NAME ______________________________ DATE ______________________________ PERIOD __________

9-1 Study Guide and Intervention
Polar Coordinates

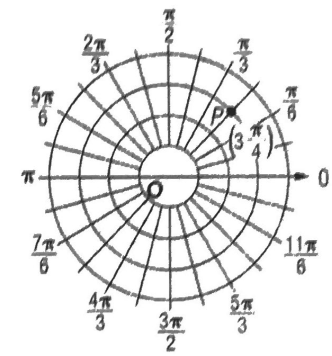
Graph Polar Coordinates A **polar coordinate system** uses distances and angles to record the position of a point. The location of a point P can be identified by polar coordinates of the form (r, θ), where r is the directed distance from the **pole**, or origin, to point P and θ is the measure of the directed angle formed by the ray from the pole to point P and the **polar axis**.

Example: Graph each point.

a. $P\left(3, \frac{\pi}{4}\right)$

Sketch the terminal side of an angle measuring $\frac{\pi}{4}$ radians in standard position.

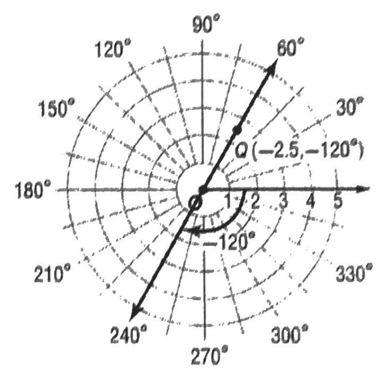
Since r is positive ($r = 3$), find the point on the terminal side of the angle that is 3 units from the pole. *Notice point P is on the third circle from the pole.*



b. $Q(-2.5, -120°)$

Negative angles are measured clockwise. Sketch the terminal side of an angle of $-120°$ in standard position.

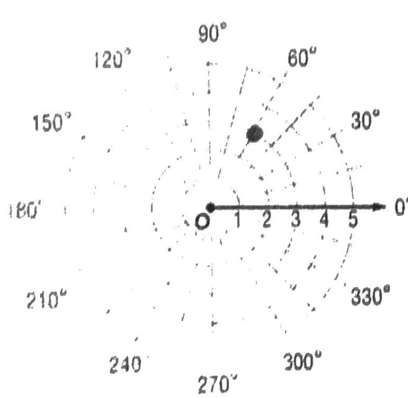
Since r is negative, extend the terminal side of the angle in the opposite direction. Find the point Q that is 2.5 units from the pole along this extended ray.



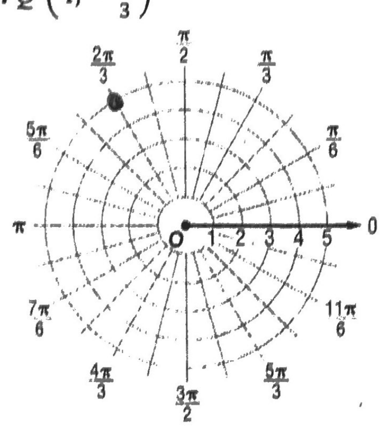
Exercises

Graph each point on a polar grid.

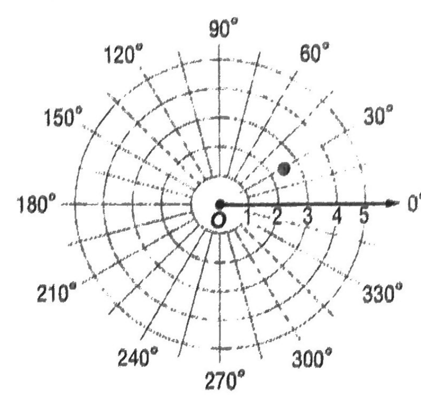
1. $R(3, 60°)$



2. $Q\left(4, -\frac{4\pi}{3}\right)$ $-\frac{8\pi}{6}$



3. $A(-2.5, -150°)$



NAME ______________________ DATE ______________ PERIOD ________

9-1 Study Guide and Intervention *(continued)*

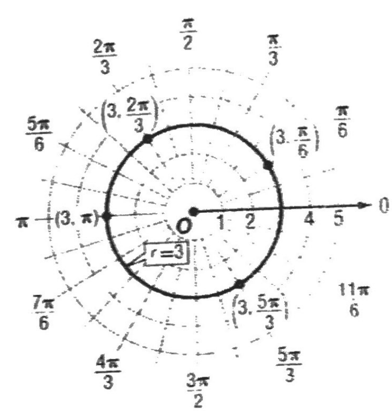
Polar Coordinates

Graphs of Polar Equations An equation expressed in terms of **polar coordinates** is called a polar equation. A polar graph is the set of all points with coordinates (r, θ) that satisfy a given polar equation. The graphs of polar equations like $r = k$ and $\theta = k$, where k is a constant, are considered basic in the polar coordinate system. The solutions of $r = k$ are ordered pairs of the form (k, θ) where θ is any real number. The solutions of $\theta = k$ are ordered pairs of the form (r, θ) where r is any real number.

Example: Graph each polar equation.

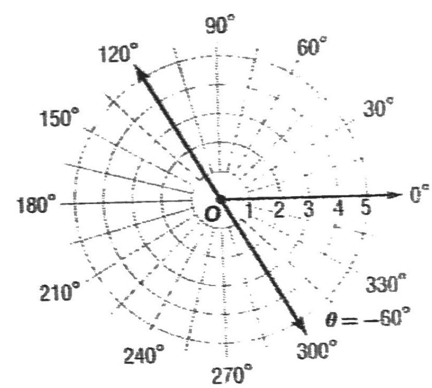
a. $r = 3$

The solutions of $r = 3$ are ordered pairs of the form $(3, \theta)$, where θ is any real number. The graph consists of all the points that are 3 units from the pole, so the graph is a circle centered at the origin with radius 3.



b. $\theta = -60°$

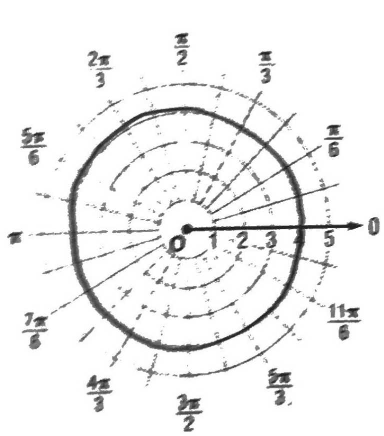
The solutions of $\theta = -60°$ are ordered pairs of the form $(r, -60°)$, where r is any real number. The graph consists of all the points on the line that make an angle of $-60°$ with the positive polar axis.



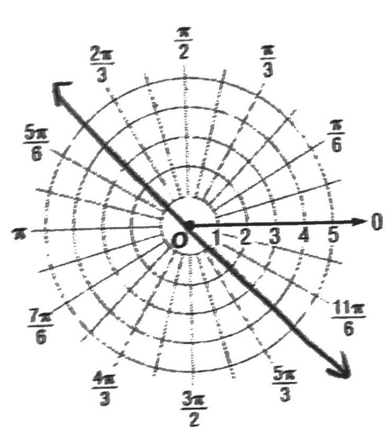
Exercises

Graph each polar equation.

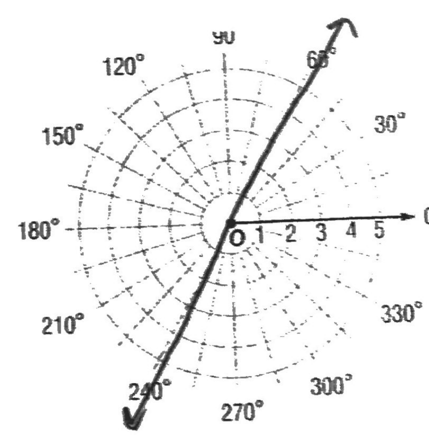
1. $r = 4$



2. $\theta = \frac{3\pi}{4}$



3. $\theta = -300°$

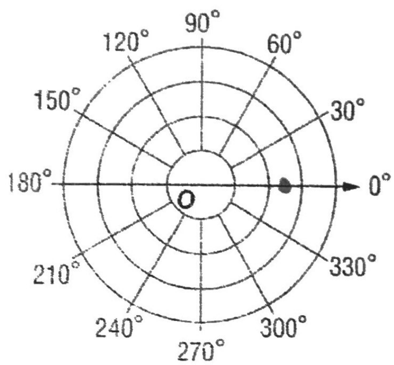


NAME ______________________ DATE ______________________ PERIOD ________

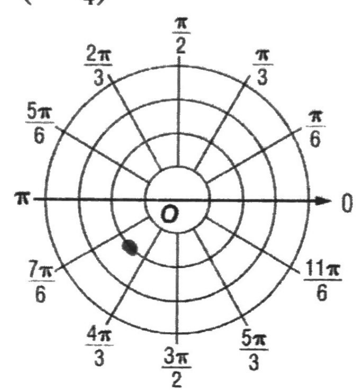
9-1 Practice
Polar Coordinates

Graph each point on a polar grid.

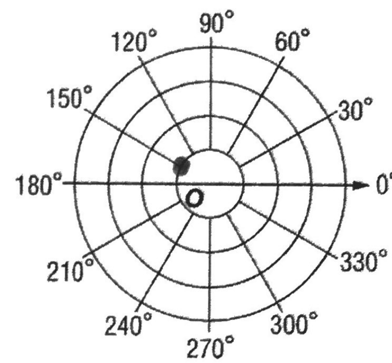
1. (2.5, 0°)



2. $\left(-2, \frac{\pi}{4}\right)$

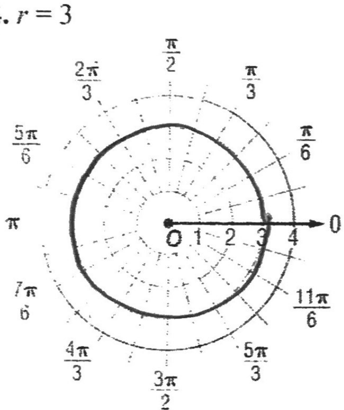


3. (–1, –30°)

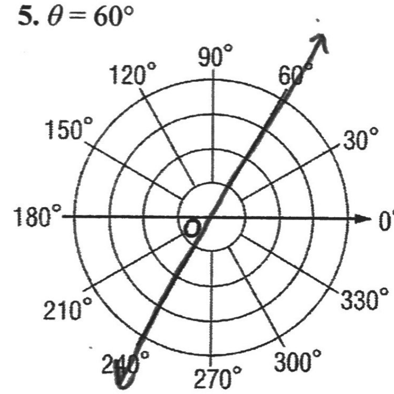


Graph each polar equation.

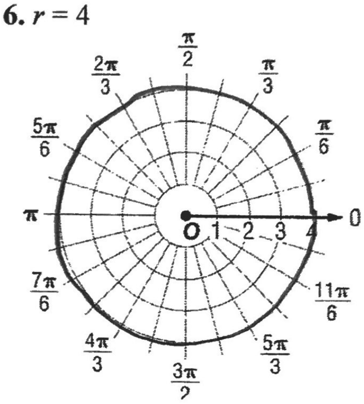
4. $r = 3$



5. $\theta = 60°$

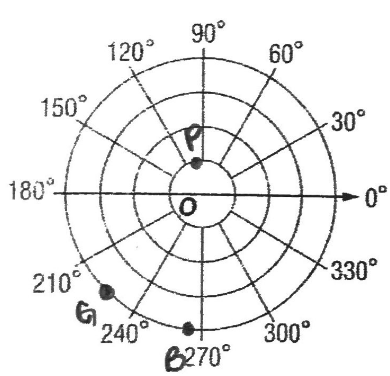


6. $r = 4$



7. LANDSCAPING A landscape architect has created a blueprint for the landscape design at a new building being constructed at a retirement community.

a. The architect has placed a gazebo at (3, –135°). Graph this point.



b. The design calls for a bench at (–4, 85°) and a pond at (1, 105°). Find the distance in feet between the pond and the bench.

distance $= \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_2 - \theta_1)} = \sqrt{(-4)^2 + (1)^2 - 2(-4)(1)\cos(85 - 105)}$

$= \boxed{4.95}$

* fun fact!
but not needed to memorize

9-2 Study Guide and Intervention

Graphs of Polar Equations

Graphs of Polar Equations A **polar graph** is the set of all points with coordinates (r, θ) that satisfy a given polar equation. The position and shape of polar graphs can be altered by multiplying or adding to either the function or θ.

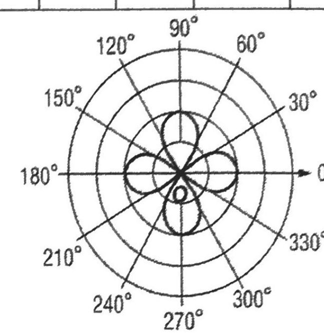
Example 1: Graph the polar equation $r = 2\cos 2\theta$.

Make a table of values on the interval $[0, 2\pi]$.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$r = 2\cos 2\theta$	2	1	0	1	–2	–1	0	1	2	1	0	–1	–2	–1	0	1	2

Graph the ordered pairs (r, θ) and connect with a smooth curve.

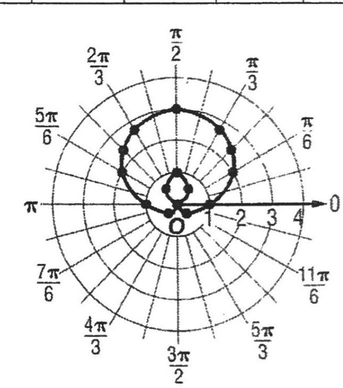
This type of curve is called a **rose**. Notice that the farthest points are 2 units from the pole and the rose has 4 petals.



Example 2: Graph the polar equation $r = 1 + 2\sin\theta$. Round each r-value to the nearest tenth.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$r = 1 + 2\sin\theta$	1	2	2.4	2.7	3	2.7	2.4	2	1	0	–0.4	–0.7	–1	–0.7	–0.4	0	1

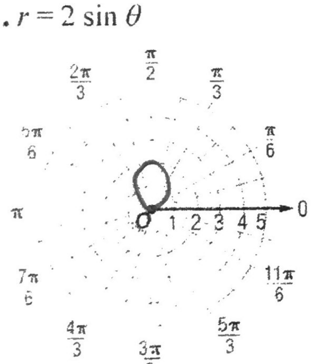
Graph the ordered pairs and connect them with a smooth curve. This type of curve is called a **limaçon**.



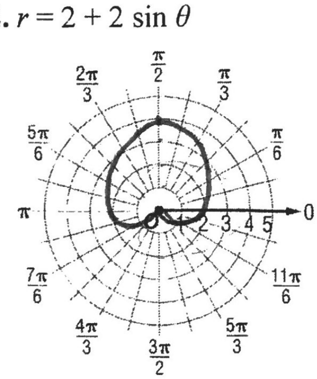
Exercises

Graph each equation by plotting points.

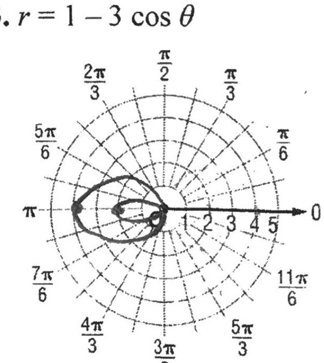
1. $r = 2\sin\theta$



2. $r = 2 + 2\sin\theta$



3. $r = 1 - 3\cos\theta$



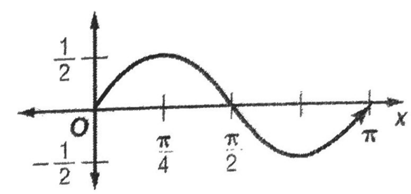
9-2 Study Guide and Intervention *(continued)*

Graphs of Polar Equations

Classic Polar Curves The graph of a polar equation is symmetric with respect to the polar axis if it is a function of $\cos\theta$, and to the line $\theta = \frac{\pi}{2}$ if it is a function of $\sin\theta$. It is symmetric to the pole if replacing (r, θ) with $(-r, \theta)$ or $(r, \pi + \theta)$ produces an equivalent equation. Knowing whether a graph is symmetric can reduce the number of points needed to sketch it.

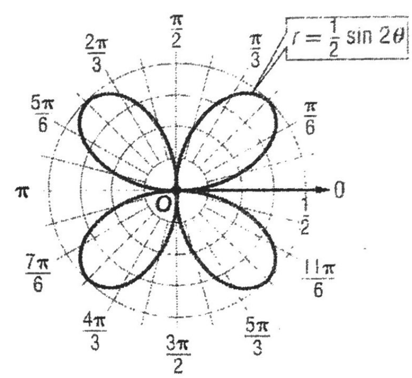
Example: Determine the symmetry, zeros, and maximum r-values of $r = \frac{1}{2}\sin 2\theta$. Then use this information to graph the function.

The function is symmetric with respect to the line $\theta = \frac{\pi}{2}$, so you can find points on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and then use line symmetry to complete the graph. To find the zeros and the maximum r-value, sketch the graph of the rectangular function $y = \frac{1}{2}\sin 2x$.



From the graph, you can see that $|y| = \frac{1}{2}$ when $x = \frac{\pi}{4}$, and $\frac{3\pi}{4}$ and $y = 0$ when $x = 0, \frac{\pi}{2}$, and π. That means that $|r|$ has a maximum value of $\frac{1}{2}$ when $\theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$ and $r = 0$ when $\theta = 0, \frac{\pi}{2}$, or π. Use these and a few additional points to sketch the graph of the function.

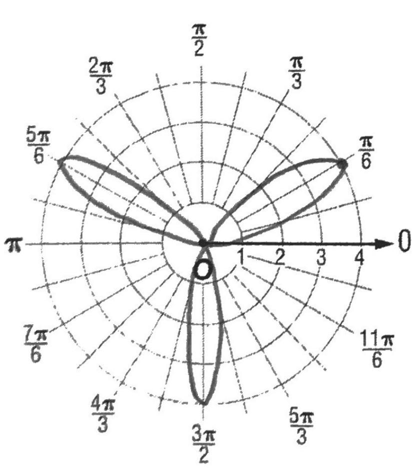
Use the axis of symmetry to complete the graph after plotting points on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.



Exercises

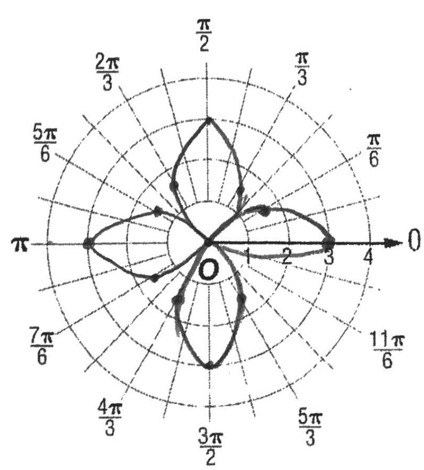
Use symmetry, zeros, and maximum r-values to graph each function.

1. $r = 4\sin 3\theta$



θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
r	0	4	0	-4	0	4	0

2. $r = 3\cos 2\theta$



θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
r	3	1.5	-1.5	-3	-1.5	1.5	3	1.5	-1.5	-3	-1.5	1.5	3

NAME ______________________ DATE ______________________ PERIOD ____________

9-2 Practice

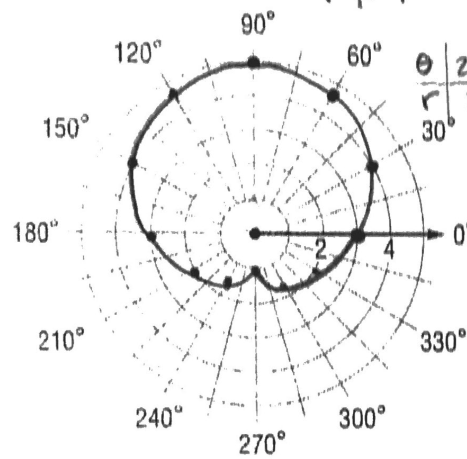
Graphs of Polar Equations

Use symmetry to graph each equation.

1. $r = 3 + 2 \sin \theta$

θ	0	30	60	90	120	150	180
r	3	4	4.7	5	4.7	4	3

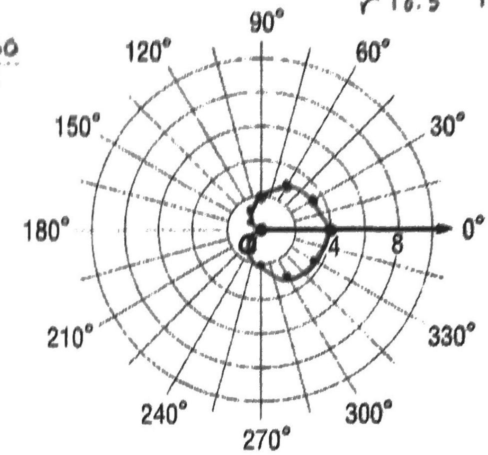
θ	210	240	270	300	330
r	2	1.3	1	1.3	2



2. $r = 2 + 2 \cos \theta$

θ	0	30	60	90	120	150	180
r	4	3.7	3	2	1	0.3	0

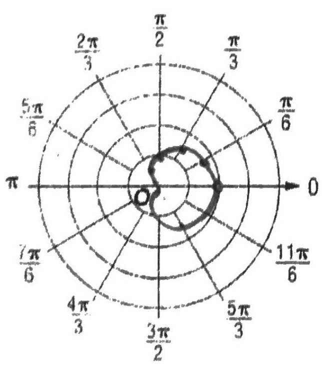
θ	210	240	270	300	330
r	0.3	1	2	3	3.7



Identify the type of curve given by each equation. Then use symmetry, zeros, and maximum *r*-values to graph the function.

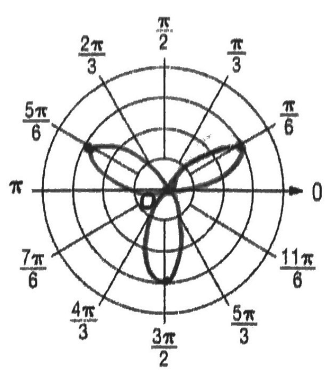
3. $r = 1 + \cos \theta$

limaçon



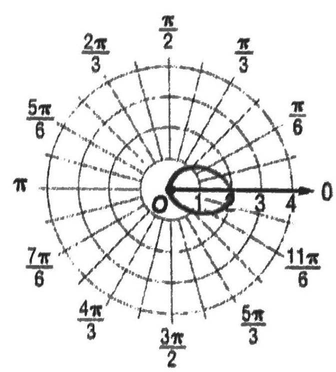
4. $r = 3 \sin 3\theta$

rose



5. $r = 2 \cos \theta$

circle



6. DESIGN Mikaela is designing a border for her stationery. Suppose she uses a rose curve. Determine an equation for designing a rose that has 8 petals with each petal 4 units long.

$y = 4\cos(4\theta)$ or $y = 4\sin(4\theta)$

NAME ______________________________ DATE ______________________________ PERIOD ____________

9-3 Study Guide and Intervention

Polar and Rectangular Forms of Equations

Polar and Rectangular Coordinates If a point P has polar coordinates (r, θ), then the rectangular coordinates (x, y) of P are given by $x = r\cos\theta$ and $y = r\sin\theta$. If a point P has rectangular coordinates (x, y), then the polar coordinates (r, θ) of P are given by $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\frac{y}{x}$, when $x > 0$, and $\theta = \tan^{-1}\frac{y}{x} + \pi$, when $x < 0$.

Example 1: Find rectangular coordinates for point P with the polar coordinates $\left(3, \frac{3\pi}{4}\right)$.

For $P\left(3, \frac{3\pi}{4}\right)$, $r = 3$ and $\theta = \frac{3\pi}{4}$. Use the conversion formulas.

$$\begin{aligned} x &= r\cos\theta & y &= r\sin\theta \\ &= 3\cos\frac{3\pi}{4} & &= 3\sin\frac{3\pi}{4} \\ &= 3\left(-\frac{\sqrt{2}}{2}\right) \text{ or } -\frac{3\sqrt{2}}{2} & &= 3\left(\frac{\sqrt{2}}{2}\right) \text{ or } \frac{3\sqrt{2}}{2} \end{aligned}$$

The rectangular coordinates of P are $\left(-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$, or $(-2.12, 2.12)$ to the nearest hundredth.

Example 2: Find two pairs of polar coordinates for point R with the rectangular coordinates $(5, -9)$.

For $R(5, -9)$, $x = 5$ and $y = -9$.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} & \theta &= \tan^{-1}\frac{y}{x} \\ &= \sqrt{5^2 + (-9)^2} & &= \tan^{-1}\frac{-9}{5} \\ &= \sqrt{106} \text{ or about } 10.30 & &= -1.06 \end{aligned}$$

One pair of polar coordinates for R is $(10.30, -1.06)$. To obtain a second pair of polar coordinates for R, you can add 2π to the θ-value. This results in $(10.30, -1.06 + 2\pi)$ or $(10.30, 5.22)$.

Exercises

Find rectangular coordinates for each point with the given polar coordinates.

1. $(20, -60°)$

$X = 20\cos(-60) = 10$

$Y = 20\sin(-60) = -17.32$

$(10, -17.32)$

2. $\left(-1, \frac{5\pi}{6}\right)$

$x = -1\cos\left(\frac{5\pi}{6}\right) = 0.866$

$y = -1\sin\left(\frac{5\pi}{6}\right) = -0.5$

$(0.866, -0.5)$

3. $(6, -30°)$

$x = 6\cos(-30) = 5.19$

$y = 6\sin(-30) = -3$

$(5.19, -3)$

4. $\left(3, \frac{\pi}{3}\right)$

$x = 3\cos(\pi/3) = 1.5$

$y = 3\sin(\pi/3) = 2.6$

$(1.5, 2.6)$

Find two pairs of polar coordinates for each point with the given rectangular coordinates if $0 \le \theta \le 2\pi$.

5. $(2, -2)$

$r = \sqrt{(2)^2 + (-2)^2} = 2\sqrt{2}$

$\theta = \tan^{-1}\left(\frac{-2}{2}\right) = -45°$

$(2\sqrt{2}, -45°)$

$(2\sqrt{2}, 315°)$

6. $(3, 5)$

$r = \sqrt{(3)^2 + (5)^2} = \sqrt{34}$

$\theta = \tan^{-1}\left(\frac{5}{3}\right) = 59.04°$

$(\sqrt{34}, 59.04°)$

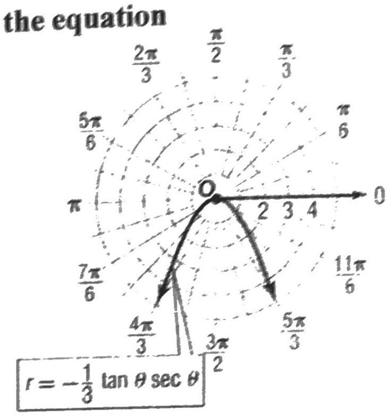
$(\sqrt{34}, -300.96°)$

9-3 Study Guide and Intervention *(continued)*

Polar and Rectangular Forms of Equations

Polar and Rectangular Equations You can also use the relationships $r^2 = x^2 + y^2$, $x = r\cos\theta$ and $y = r\sin\theta$. and $\tan\theta = \frac{y}{x}$ to convert between rectangular equations and polar equations.

Example 1: Identify the graph of the rectangular equation $y = -3x^2$. Then write the equation in polar form. Support your answer by graphing the polar form of the equation.

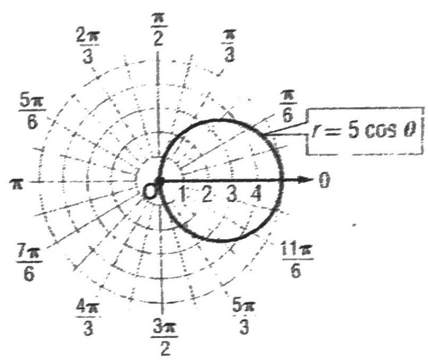


The graph of $y = -3x^2$ is a parabola with vertex at the origin that opens down.

$y = -3x^2$	Original equation
$r\sin\theta = -3(r\cos\theta)^2$	$x = r\cos\theta$ and $y = r\sin\theta$
$r\sin\theta = -3r^2\cos^2\theta$	Multiply.
$\frac{\sin\theta}{-3\cos^2\theta} = r$	Divide each side by $-3r\cos^2\theta$.
$-\frac{1}{3}\tan\theta\sec\theta = r$	Quotient and Reciprocal Identities

The graph of the polar equation $r = -\frac{1}{3}\tan\theta\sec\theta$ is a parabola with vertex at the pole that opens down.

Example 2: Write the polar equation $r = 5\cos\theta$ in rectangular form and then identify its graph. Support your answer by graphing the polar form of the equation.



$r = 5\cos\theta$	Original equation
$r^2 = 5r\cos\theta$	Multiply each side by r.
$x^2 + y^2 = 5x$	$r^2 = x^2 + y^2$ and $r\cos\theta = x$
$x^2 - 5x + y^2 = 0$	Subtract $5x$ from each side.

Because in standard form this equation is $(x - 2.5)^2 + y^2 = 6.25$, you can identify the graph of this equation as a circle centered at (2.5, 0) with radius 2.5, as supported by the graph of $r = 5\cos\theta$.

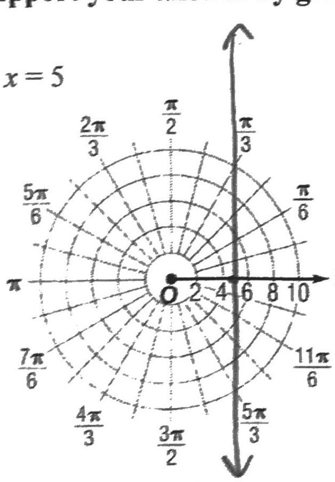
Exercises

Identify the graph of each rectangular equation. Then write the equation in polar form. Support your answer by graphing the polar form of the equation.

1. $x = 5$

$r\cos\theta = 5$

$r = \frac{5}{\cos\theta}$



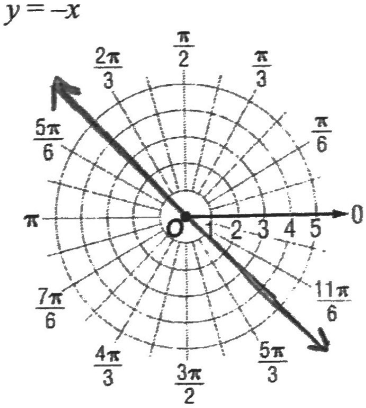
θ	0	30	60	90
r	5	5.77	10	und.

2. $y = -x$

$r\sin\theta = -r\cos\theta$

$\sin\theta = -\cos\theta$

$\tan\theta = -1$



NAME _______________ DATE _______________ PERIOD _______________

9-3 Practice

Polar and Rectangular Forms of Equations

Find the rectangular coordinates for each point with the given polar coordinates.

1. (6, 120°)
$x = 6\cos 120 = -3$
$y = 6\sin 120 = 5.2$
$(-3, 5.2)$

2. (–4, 45°)
$x = -4\cos(45) = -2.8$
$y = -4\sin(45) = -2.8$
$(-2.8, -2.8)$

3. $\left(4, \frac{\pi}{6}\right)$
$x = 4\cos(\pi/6) = 3.46$
$y = 4\sin(\pi/6) = 2$
$(3.46, 2)$

Find two pairs of polar coordinates for each point with the given rectangular coordinates if $0 \le \theta < 2\pi$.

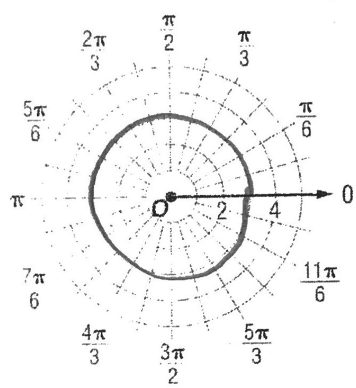
4. (2, 2)
$(2\sqrt{2}, 45°), (2\sqrt{2}, -315°)$
$r = \sqrt{8} = 2\sqrt{2}$ $\theta = \tan^{-1}\left(\frac{2}{2}\right) = 45°$

5. (2, –3)
$(\sqrt{13}, -56.3°), (\sqrt{13}, 303.7°)$
$r = \sqrt{13}$ $\theta = \tan^{-1}\left(\frac{-3}{2}\right) = -56.3°$

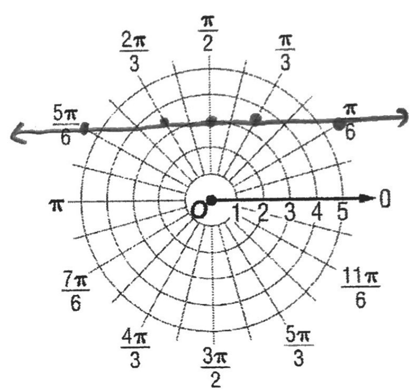
6. $(-3, \sqrt{3})$
$(2\sqrt{3}, 150°), (2\sqrt{3}, -210°)$
$r = \sqrt{12} = 2\sqrt{3}$ $\theta = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) + 180 = 150°$

Identify the graph of each rectangular equation. Then write the equation in polar form. Support your answer by graphing the polar form of the equation.

7. $x^2 + y^2 = 9$
$(r\cos\theta)^2 + (r\sin\theta)^2 = 9$
$r^2\cos^2\theta + r^2\sin^2\theta = 9$
$r^2(\cos^2\theta + \sin^2\theta) = 9$
$r^2 = 9$
$r = 3$



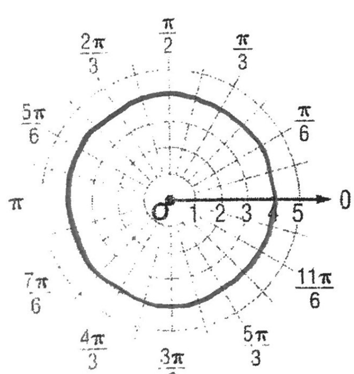
8. $y = 3$
$r\sin\theta = 3$
$r = \frac{3}{\sin\theta}$



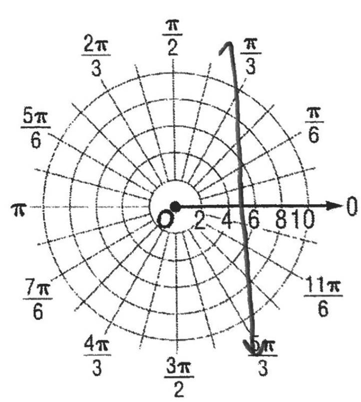
θ	0	30	60	90	120	150
r	err.	6	3.5	3	3.5	6

Write each equation in rectangular form and then identify its graph. Support your answer by graphing the polar form of the equation.

9. $r = 4$
$4 = \sqrt{x^2 + y^2}$
$16 = x^2 + y^2$



10. $r\cos\theta = 5$
$x = 5$



11. SURVEYING A surveyor records the polar coordinates of the location of a landmark as (40, 62°). What are the rectangular coordinates?
$(18.8, 35.3)$
$x = 40\cos(62) = 18.8$
$y = 40\sin(62) = 35.3$