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7-5 Study Guide and Intervention

Parametric Equations

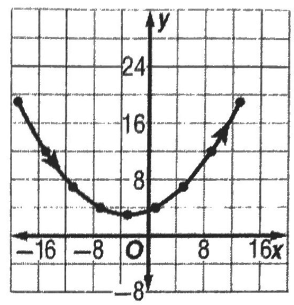
Graph Parametric Equations Parametric equations are used to describe the horizontal and vertical components of an equation. **Parameters** are arbitrary values, usually time or angle measurement.

Example 1: Sketch the curve given by the parametric equations $x = -3 + 4t$ and $y = t^2 + 3$ over the interval $-4 \le t \le 4$.

Make a table of values for $-4 \le t \le 4$.

t	x	y	t	x	y
−4	−19	19	0	−3	3
−3	−15	12	1	1	4
−2	−11	7	2	5	7
−1	−7	4	3	9	12
0	−3	3	4	13	19

Plot the (x, y) coordinates for each t–value and connect the points to form a smooth curve.



Example 2: Write $x = 4t - 2$ and $y = t^2 + 1$ in rectangular form.

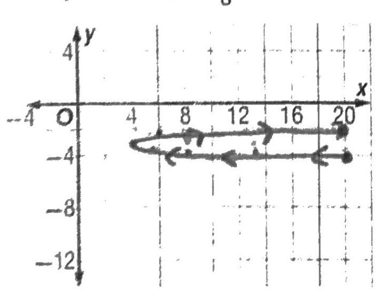
$x = 4t - 2$	Parametric equation for x
$\frac{x+2}{4} = t$	Solve for t.
$y = \left(\frac{x+2}{4}\right)^2 + 1$	Substitute $\frac{x+2}{4}$ for t in the equation for y.
$= \frac{x^2 + 4x + 4}{16} + 1$	Square $x + 2 - 4$.
$= \frac{x^2}{16} + \frac{x}{4} + \frac{5}{4}$	Simplify.

The rectangular equation is $y = \frac{x^2}{16} + \frac{x}{4} + \frac{5}{4}$.

Exercises

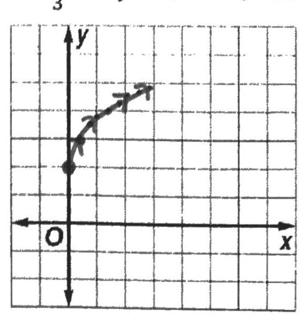
Sketch the curve given by each pair of parametric equations over the given interval.

1. $x = t^2 + 4$ and $y = \frac{t}{6} - 3$; $-4 \le t \le 4$



t	x	y
-4	20	-3.667
-3	13	-3.5
-2	8	-3.33
-1	5	-3.167
0	4	-3
1	5	-2.833
2	8	-2.667
3	13	-2.5
4	20	-2.33

2. $x = \frac{t}{3}$ and $y = \sqrt{t} + 2$; $0 \le t \le 8$



t	x	y
0	0	2
1	0.33	3
2	0.667	3.4142
3	1	3.7
4	1.33	4
5	1.667	4.2361
6	2	4.4495
7	2.33	4.6458
8	2.667	4.8284

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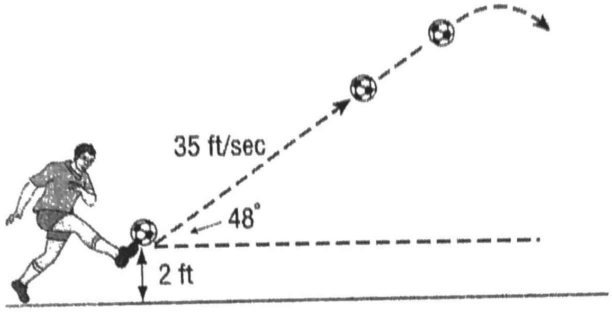
7-5 Study Guide and Intervention *(continued)*

Parametric Equations

Projectile Motion Parametric equations are often used to simulate projectile motion. For an object launched at an angle θ with the horizontal at an initial velocity v_0, where g is the gravitational constant, t is time, and h_0 is the initial height, the horizontal distance x can be found by $x = tv_0 \cos \theta$ and the vertical position y by $y = tv_0 \sin \theta - \frac{1}{2}gt^2 + h_0$.

Example: Luigi is kicking a soccer ball. He kicks the ball with an initial velocity of 35 feet per second at an angle of 48° with the horizontal. The ball is 2 feet above the ground when he kicks it. How far will the ball travel horizontally before it hits the ground?

Step 1 Make a diagram of the situation.



Step 2 Write a parametric equation for the vertical position of the ball.

$y = tv_0 \sin \theta - \frac{1}{2}gt^2 + h_0$ — Parametric equation for vertical position

$= t(35) \sin (48) - \frac{1}{2}(32)t^2 + 2$ — $v_0 = 35$, $\theta = 48°$, $g = 32$, and $h_0 = 2$

Step 3 Graph the equation for the vertical position and the line $y = 0$. Use **5: INTERSECT** on the **CALC** menu of a calculator to find the point of intersection of the curve with $y = 0$. The value is about 1.7 seconds. You could also use **2: ZERO** and not graph $y = 0$.

Step 4 Determine the horizontal position of the ball at 1.7 seconds.

$x = tv_0 \cos \theta$ — Parametric equation for horizontal position

$= 1.7(35) \cos 48$ — $v_0 = 35$, $\theta = 48°$, and $t = 1.7$

≈ 39.8 — Use a calculator.

The ball will travel about 39.8 feet before it hits the ground.

Exercises

1. Julie is throwing a ball at an initial velocity of 28 feet per second and an angle of 56° with the horizontal from a height of 4 feet. How far away will the ball land?

$x = t(28)\cos(56) \rightarrow t = \frac{x}{15.67}$

$y = \left(\frac{x}{15.67}\right)(28)\sin(56) - \frac{1}{2}(32)\left(\frac{x}{15.67}\right)^2 + 4 = 0$ ← x-int on graph

$\boxed{x = 25.17 \text{ ft}}$

2. Jerome hits a tennis ball at an initial velocity of 38 feet per second and an angle of 42° with the horizontal from a height of 1.5 feet. How far away will the ball land if it is not hit by his opponent?

$x = t(38)\cos(42°) \rightarrow t = \frac{x}{28.24}$

$y = \left(\frac{x}{28.24}\right)(38)\sin(42) - \frac{1}{2}(32)\left(\frac{x}{28.24}\right)^2 + 1.5 = 0$

$\boxed{x = 46.48 \text{ ft}}$

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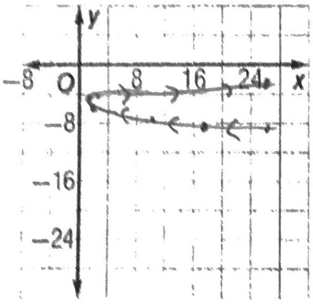
7-5 Practice
Parametric Equations

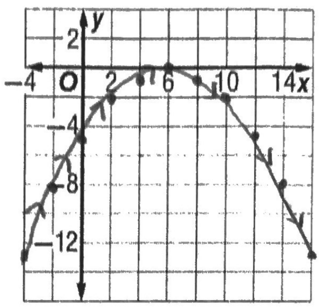
Sketch the curve given by each pair of parametric equations over the given interval.

1. $x = t^2 + 1$ and $y = \frac{t}{2} - 6$; $-5 \le t \le 5$



t	x	y
-5	26	-8.5
-4	17	-8
-3	10	-7.5
-2	5	-7
-1	2	-6.5
0	1	-6
1	2	-5.5
2	5	-5
3	10	-4.5
4	17	-4
5	26	-3.5

2. $x = 2t + 6$ and $y = -\frac{t^2}{2}$; $-5 \le t \le 5$



t	x	y
-5	-4	-12.5
-4	-2	-8
-3	0	-4.5
-2	2	-2
-1	4	-0.5
0	6	0
1	8	-0.5
2	10	-2
3	12	-4.5
4	14	-8
5	16	-12.5

Write each pair of parametric equations in rectangular form.

3. $x = 2t + 3$, $y = t - 4$

$\frac{x-3}{2} = t$ $\boxed{y = \frac{x-3}{2} - 4}$

4. $x = t + 5$, $y = -3t^2$

$x - 5 = t$ $\boxed{y = -3(x-5)^2}$

5. $x = 3 \sin \theta$, $y = 2 \cos \theta$

$\frac{x}{3} = \sin\theta$ $\frac{y}{2} = \cos\theta$ $\boxed{\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1}$

6. $y = 4 \sin \theta$, $x = 5 \cos \theta$

$\frac{y}{4} = \sin\theta$ $\frac{x}{5} = \cos\theta$ $\boxed{\left(\frac{x}{5}\right)^2 + \left(\frac{y}{4}\right)^2 = 1}$

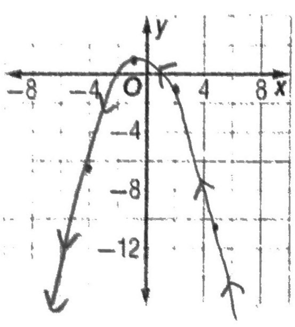
Use each parameter to write the parametric equations that can represent each equation. Then graph the equation, indicating the speed and orientation.

7. $t = \frac{2 - x}{3}$ for $y = \frac{3 - x^2}{2}$

$3t = 2 - x$

$\boxed{x = 2 - 3t}$

$\boxed{y = \frac{3 - (2-3t)^2}{2}}$



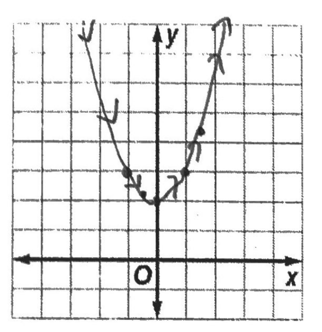
t	x	y
-1	5	-11
0	2	-0.5
1	-1	1
2	-4	-6.5

8. $t = 4x - 1$ for $y = x^2 + 2$

$t + 1 = 4x$

$\boxed{\frac{t+1}{4} = x}$

$\boxed{y = \left(\frac{t+1}{4}\right)^2 + 2}$



t	x	y
-5	-1	3
-3	-0.5	2.25
-1	0	2
3	1	3
5	1.5	4.25

9. **MODEL ROCKETRY** Manuel launches a toy rocket from ground level with an initial velocity of 80 feet per second at an angle of 80° with the horizontal.

a. Write parametric equations to represent the path of the rocket.

$x = t(80)\cos(80°)$ $y = t(80)\sin(80) - \frac{1}{2}(32)(t^2)$

b. How long will it take the rocket to travel 10 feet horizontally from is starting point? What will be its vertical distance at that point?

$10 = t(80)\cos(80)$

$\boxed{0.7198 \text{ sec} = t}$

$y = 0.7198(80)\sin(80) - \frac{1}{2}(32)(0.7198)^2$

$\boxed{y = 48.4194 \text{ ft}}$

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8-1 Study Guide and Intervention

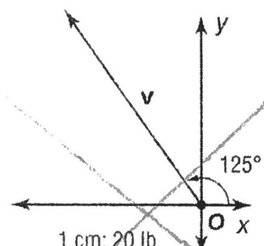
Introduction to Vectors

Geometric Vectors A vector is a quantity that has both magnitude and direction. The magnitude of a vector is the length of a directed line segment, and the direction of a vector is the directed angle between the positive x-axis and the vector. When adding or subtracting vectors, you can use the parallelogram or triangle method to find the resultant.

Example: Use a ruler and a protractor to draw an arrow diagram for each quantity described. Include a scale on each diagram.

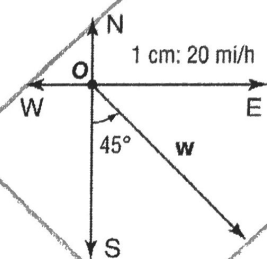
a. v = 60 pounds of force at 125° to the horizontal

Using a scale of 1 cm: 20 lb, draw and label a $60 \div 20$ or 3-centimeter arrow in standard position at a 125° angle to the x-axis.



b. w = 55 miles per hour at a bearing of S45°E

Using a scale of 1 cm.: 20 mi/h, draw and label a $55 \div 20$ or 2.75-centimeter arrow 45° east of south.



Exercises

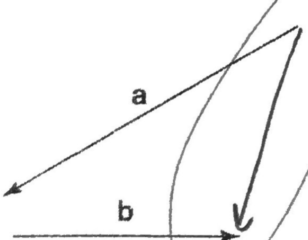
Use a ruler and a protractor to draw an arrow diagram for each quantity described. Include a scale on each diagram.

1. **r** = 30 meters at a bearing of N45°W

2. **t** = 150 yards at 40° to the horizontal

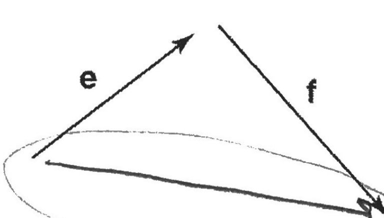
Find the resultant of each pair of vectors using either the triangle or parallelogram method. State the magnitude of the resultant in centimeters and its direction relative to the horizontal.

3.



* tip to tail

4.



do not need to use a ruler & protractor to find magnitude

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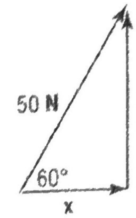
8-1 Study Guide and Intervention *(continued)*
Introduction to Vectors

Vector Applications Vectors can be resolved into horizontal and vertical components.

Example: Suppose Jamal pulls on the ends of a rope tied to a dinghy with a force of 50 Newtons at an angle of 60° with the horizontal.



a. Draw a diagram that shows the resolution of the force Jamal exerts into its rectangular components.

Jamal's pull can be resolved into a horizontal pull **x** forward and a vertical pull **y** upward as shown.

b. Find the magnitudes of the horizontal and vertical components of the force.

The horizontal and vertical components of the force form a right triangle. Use the sine or cosine ratios to find the magnitude of each force.

$\cos 60° = \frac{\lvert x\rvert}{50}$	Right triangle definitions of cosine and sine	$\sin 60° = \frac{\lvert y\rvert}{50}$
$\lvert x\rvert = 50 \cos 60°$	Solve for *x* and *y*.	$\lvert y\rvert = 50 \sin 60°$
$\lvert x\rvert = 25$	Use a calculator.	$\lvert y\rvert \approx 43.3$

The magnitude of the horizontal component is about 25 Newtons, and the magnitude of the vertical component is about 43 Newtons.

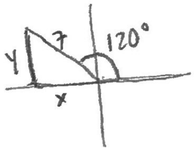
Exercises

Draw a diagram that shows the resolution of each vector into its rectangular components. Then find the magnitudes of the vector's horizontal and vertical components.

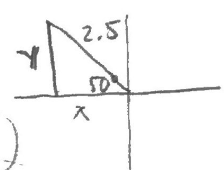
1. 7 inches at a bearing of 120° from the horizontal

$X = |7\cos 120°| = |-3.5\ N| = 3.5N$

$Y = 7\sin 120° = 6.06N$



2. 2.5 centimeters per hour at a bearing of N50°W



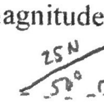
$X = 2.5\cos 50° = 1.6N$

$Y = 2.5\sin 50° = 1.92N$

3. YARDWORK Nadia is pulling a tarp along level ground with a force of 25 pounds directed along the tarp. If the tarp makes an angle of 50° with the ground, find the horizontal and vertical components of the force. What is the magnitude and direction of the resultant?

$X = 25\cos 50° = 16.1N$ $\quad Y = 25\sin 50° = 19.2N$

magnitude = 25 N
direction = 50° above horizontal

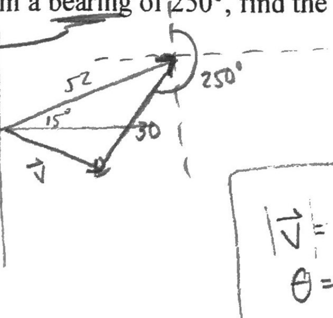


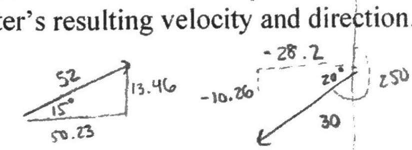
4. TRANSPORTATION A helicopter is moving 15° north of east with a velocity of 52 km/h. If a 30-kilometer per hour wind is blowing from a bearing of 250°, find the helicopter's resulting velocity and direction.

$X = 50.23 + -28.2 = 22.03$

$Y = 13.46 + -10.26 = 3.2$

0° = N
90° = E
180° = S
270° = W





$|\vec{v}| = \sqrt{22.03^2 + 3.2^2} = 22.26$ mph

$\theta = \tan^{-1}\left(\frac{3.2}{22.03}\right) = E8.26°S$

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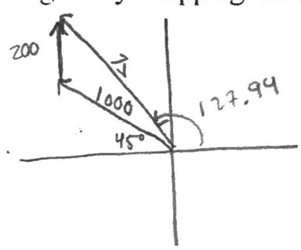
8-1 Practice
Introduction to Vectors

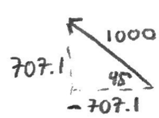
Use a ruler and a protractor to draw an arrow diagram for each quantity described. Include a scale on each diagram.

1. $\mathbf{r}$ = 60 meters at a bearing of N45°E

2. $\mathbf{t}$ = 100 pounds of force at 60° to the horizontal

3. GROCERY SHOPPING Caroline walks 45° north of west for 1000 feet and then walks 200 feet due north to go grocery shopping. How far and at what north of west quadrant bearing is Caroline from her apartment?





$x = -707.1 + 0 = -707.1$

$y = 707.1 + 200 = 907.1$

$|\vec{v}| = \sqrt{(-707.1)^2 + (907.1)^2}$

$\boxed{|\vec{v}| = 1150.14 \text{ ft}}$

$\boxed{\theta = W52.06N}$

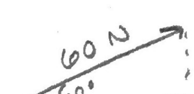
$\theta = \tan^{-1}\left(\frac{907.1}{-707.1}\right) + 180$

x is negative

$\theta = 127.94°$

4. CONSTRUCTION Roland is pulling a crate of construction materials with a force of 60 Newtons at an angle of 42° with the horizontal.

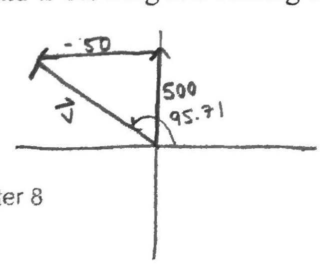
a. Draw a diagram that shows the resolution of the force Roland exerts into its rectangular components.



b. Find the magnitudes of the horizontal and vertical components of the force.

$x = 60\cos 60° = 30 \text{ N}$ $\quad$ $y = 60\sin 60° = 52 \text{ N}$

5. AVIATION An airplane is flying with an airspeed of 500 miles per hour on a heading due north. If a 50-mile per hour wind is blowing at a bearing of 270°, determine the velocity and direction of the plane relative to the ground.



$|\vec{v}| = \sqrt{50^2 + 500^2}$

$\boxed{|\vec{v}| = 502.5 \text{ mph}}$

$\boxed{\theta = W84.3°N}$

$\theta = \tan^{-1}\left(\frac{500}{-50}\right) + 180 = 95.71°$