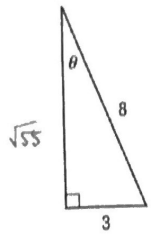
NAME _____ DATE _____ PERIOD _____

4-1 Practice
Right Triangle Trigonometry

Find the exact values of the six trigonometric functions of θ.

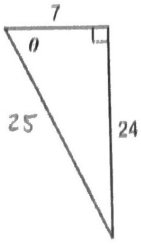
1.



$\sin\theta = \frac{3}{8}$ $\cos\theta = \frac{\sqrt{55}}{8}$ $\tan\theta = \frac{3}{\sqrt{55}} \cdot \frac{\sqrt{55}}{\sqrt{55}} = \frac{3\sqrt{55}}{55}$ $\csc\theta = \frac{8}{3}$ $\sec\theta = \frac{8}{\sqrt{55}} \cdot \frac{\sqrt{55}}{\sqrt{55}} = \frac{8\sqrt{55}}{55}$ $\cot\theta = \frac{\sqrt{55}}{3}$

$3^2 + b^2 = 8^2$ $b^2 = 55$

2.

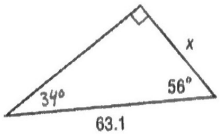


$\sin\theta = \frac{24}{25}$ $\cos\theta = \frac{7}{25}$ $\tan\theta = \frac{24}{7}$ $\csc\theta = \frac{25}{24}$ $\sec\theta = \frac{25}{7}$ $\cot\theta = \frac{7}{24}$

$7^2 + 24^2 = c^2$ $625 = c^2$

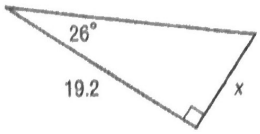
Find the value of x. Round to the nearest tenth, if necessary.

3.



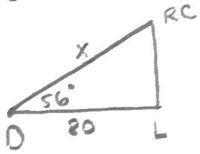
$\frac{\sin 34}{x} = \frac{\sin 90}{63.1}$ $\frac{x\sin 90}{\sin 90} = \frac{63.1\sin 34}{\sin 90}$ $x = 35.3$

4.



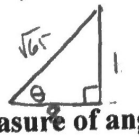
$\tan 26 = \frac{x}{19.2}$ $19.2\tan 26 = x$ $9.4 = x$

5. On a college campus, the library is 80 yards due east of the dormitory and the recreation center is due north of the library. The college is constructing a sidewalk from the dormitory to the recreation center. The sidewalk will be at a 56° angle with the current sidewalk between the dormitory and the library. To the nearest yard, how long will the new sidewalk be? $\cos 56 = \frac{80}{x}$ $x = \frac{80}{\cos 56}$ 143 yds



6. If $\cot A = 8$, find the exact values of the remaining trigonometric functions for the acute angle A.

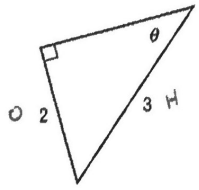
$\cot = \frac{1}{\tan}$ $\tan = \frac{1}{8}$



$1^2 + 8^2 = c^2$ $65 = c^2$ $\sin\theta = \frac{1}{\sqrt{65}} = \frac{\sqrt{65}}{65}$ $\cos\theta = \frac{8}{\sqrt{65}} = \frac{8\sqrt{65}}{65}$ $\tan\theta = \frac{1}{8}$ $\csc = \sqrt{65}$ $\sec = \frac{\sqrt{65}}{8}$

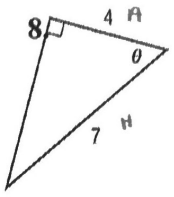
Find the measure of angle θ. Round to the nearest degree, if necessary.

7.



$\sin^{-1}\left(\frac{2}{3}\right) = 42°$

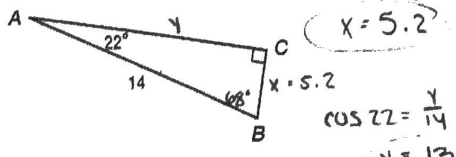
8.



$\cos^{-1}\left(\frac{4}{7}\right) = 55°$

Solve each triangle. Round side measures to the nearest tenth and angle measures to the nearest degree.

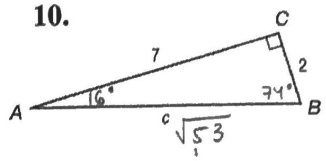
9.



$\sin 22 = \frac{x}{14}$ $x = 5.2$

$\cos 22 = \frac{y}{14}$ $y = 13$

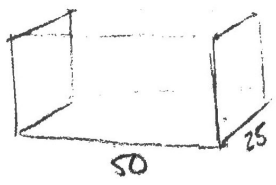
10.

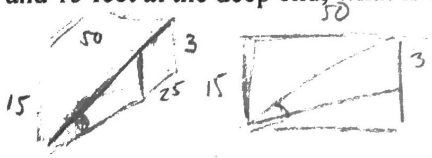


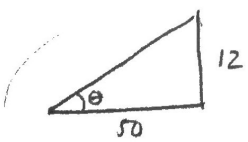
$2^2 + 7^2 = c^2$ $4 + 49 = c^2$ $53 = c^2$

$\sin^{-1}\left(\frac{7}{\sqrt{53}}\right) = B$ $B = 74°$

11. SWIMMING The swimming pool at Perris Hill Plunge is 50 feet long and 25 feet wide. If the bottom of the pool is slanted so that the water depth is 3 feet at the shallow end and 15 feet at the deep end, what is the angle of elevation at the bottom of the pool?







$\tan\theta = \frac{12}{50}$ $\theta = \tan^{-1}\left(\frac{12}{50}\right) = 13.5°$

NAME _____ DATE _____ PERIOD _____

Practice

Degrees and Radians

Write each decimal degree measure in DMS form and each DMS measure in decimal degree form to the nearest thousandth.

1. 28.955 $28 + 0.955(60) = 28 + 57.3$ $28 + 57 + 0.3(60) = 28 + 57 + 18$ $28° 57' 18''$

2. −57.3278 $= -57 + 0.3278(60) = -57 + 19.668$ $-57° 19' 40.08''$ $= -57 + 19 + 0.668(60)$ $= -57 + 19 + 40.08$

3. 32 $\frac{28'}{60} \frac{10''}{3600} = 32.4694°$

4. −73 $\frac{14'}{60} \frac{35''}{3600} = -73.243°$

Write each degree measure in radians as a multiple of π and each radian measure in degrees.

5. $25° \cdot \frac{\pi}{180} = \frac{5\pi}{36}$

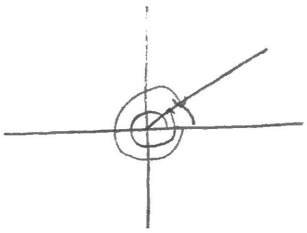
6. $130° \cdot \frac{\pi}{180} = \frac{13\pi}{18}$

7. $\frac{3\pi}{4} \cdot \frac{180}{\pi} = 135°$

8. $\frac{5\pi}{3} \cdot \frac{180}{\pi} = 300°$

Identify all angles that are coterminal with the given angle. Then find and draw one positive and one negative angle coterminal with the given angle.

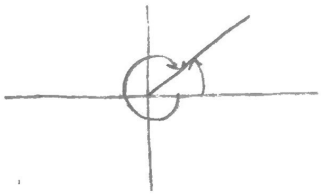
9. 43° $43° + 360°n$



$43 + 360 = 403°$

$360 - 43 = -317°$

10. $-\frac{7\pi}{4}$ $-\frac{7\pi}{4} - 2\pi n$



$\frac{\pi}{4}$

$-\frac{7\pi}{4} - 2\pi = -\frac{15\pi}{4}$

$s = r\theta$

Find the length of the intercepted arc with the given central angle measure in a circle of the given radius. Round to the nearest tenth.

11. 30°, $r = 8$ yd $s = \left(\frac{\pi}{6}\right)(8) = 4.2$ yd $= \frac{\pi}{6}$

12. $\frac{7\pi}{6}$, $r = 10$ in.

Find the rotation in revolutions per minute given the angular speed and the radius given the linear speed and the rate of rotation.

13. $\omega = \frac{4}{5}\pi$ rad/s

14. $V = 32$ m/s, 100 rev/min

15. On a game show, a contestant spins a wheel. The angular speed of the wheel was $\omega = \frac{\pi}{3}$ radians per second. If the wheel maintained this rate, what would be the rotation in revolutions per minute?

πr^2

Find the area of each sector.

16. $\theta = \frac{\pi}{6}$, $r = 14$ in.

$\frac{\frac{\pi}{6}}{2\pi} \cdot \pi(14)^2 = 51.3 \text{ in}^2$

17. $\theta = \frac{7\pi}{4}$, $r = 4$ m

$\frac{\frac{7\pi}{4}}{2\pi} \cdot \pi(4)^2 = 44 \text{ m}^2$

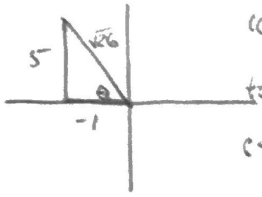
NAME _____ DATE _____ PERIOD _____

4-3 Practice

Trigonometric Functions on the Unit Circle

The given point lies on the terminal side of an angle θ in standard position. Find the values of the six trigonometric functions of θ.

1. $(-1, 5)$



$\sin\theta = \frac{5}{\sqrt{26}} = \frac{5\sqrt{26}}{26}$

$\cos\theta = -\frac{1}{\sqrt{26}} = -\frac{\sqrt{26}}{26}$

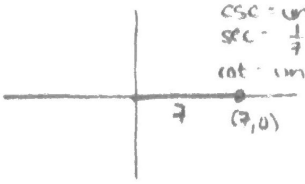
$\tan\theta = -\frac{5}{1}$

$\csc\theta = \frac{\sqrt{26}}{5}$

$\sec\theta = -\frac{\sqrt{26}}{1}$

$\cot\theta = -\frac{1}{5}$

2. $(7, 0)$



$\sin\theta = 0$

$\cos\theta = 1$

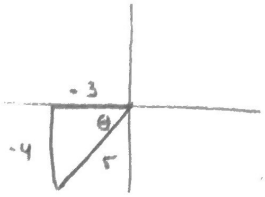
$\tan\theta = 0$

$\csc\theta = \text{und.}$

$\sec\theta = \frac{1}{1}$

$\cot\theta = \text{und.}$

3. $(-3, -4)$

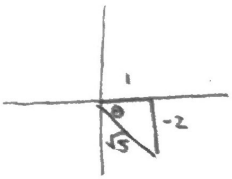


$\sin\theta = -\frac{4}{5}$ $\quad \csc\theta = -\frac{5}{4}$

$\cos\theta = -\frac{3}{5}$ $\quad \sec\theta = -\frac{5}{3}$

$\tan\theta = \frac{4}{3}$ $\quad \cot\theta = \frac{3}{4}$

4. $(1, -2)$

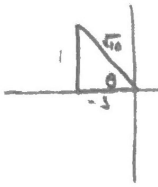


$\sin\theta = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$ $\quad \csc\theta = -\frac{\sqrt{5}}{2}$

$\cos\theta = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ $\quad \sec\theta = \frac{\sqrt{5}}{1}$

$\tan\theta = -\frac{2}{1}$ $\quad \cot\theta = -\frac{1}{2}$

5. $(-3, 1)$



$\sin\theta = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

$\cos\theta = -\frac{3}{\sqrt{10}} = -\frac{3\sqrt{10}}{10}$

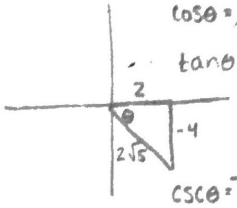
$\tan\theta = -\frac{1}{3}$

$\csc\theta = \sqrt{10}$

$\sec\theta = -\frac{\sqrt{10}}{3}$

$\cot\theta = -3$

6. $(2, -4)$



$\sin\theta = \frac{-4}{2\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

$\cos\theta = \frac{2}{2\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\tan\theta = -\frac{4}{2} = -2$

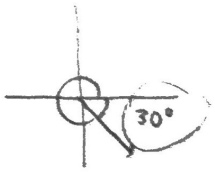
$\csc\theta = -\frac{2\sqrt{5}}{4} = -\frac{\sqrt{5}}{2}$

$\sec\theta = \frac{\sqrt{5}}{1}$

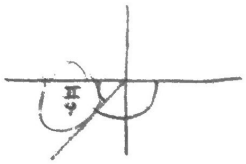
$\cot\theta = -\frac{1}{2}$

Sketch each angle. Then find its reference angle.

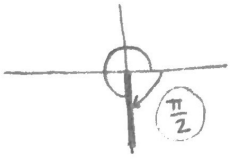
7. $330°$



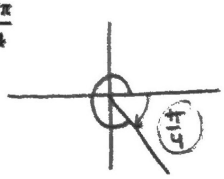
8. $-\frac{3\pi}{4}$



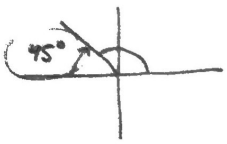
9. $\frac{9\pi}{6}$



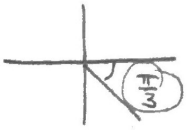
10. $\frac{7\pi}{4}$



11. $135°$



12. $-\frac{\pi}{3}$



Find the exact value of each expression. If undefined, write *undefined*.

13. $\csc 90° = \frac{1}{\sin 90°} = \frac{1}{1} = 1$

14. $\tan 270° = \frac{\sin(270)}{\cos(270)} = \frac{-1}{0} = \text{und.}$

15. $\sin(-90°) = \sin(270°) = -1$

16. $\cos\frac{3\pi}{2} = 0$

17. $\text{Sec}\left(-\frac{\pi}{4}\right) = \frac{1}{\cos\left(\frac{3\pi}{4}\right)} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

18. $\text{Cot}\frac{5\pi}{6} = \frac{\cos\left(\frac{5\pi}{6}\right)}{\sin\left(\frac{5\pi}{6}\right)} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$

19. PENDULUMS The angle made by the swing of a pendulum and its vertical resting position can be modeled by $\theta = 3\cos \pi t$, where t is time measured in seconds and θ is measured in radians. What is the angle made by the pendulum after 6 seconds?

$\theta = 3\cos(\pi \cdot 6)$

$\theta = 3\cos(6\pi)$

$\theta = 3(1) = 3 \text{ rad}$

NAME _____ DATE _____ PERIOD _____

4-4 Practice

Graphing Sine and Cosine Functions

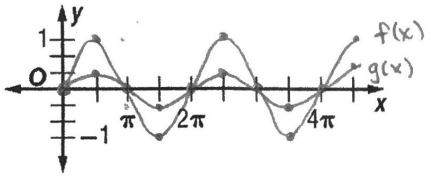
Describe how the graphs of $f(x)$ and $g(x)$ are related. Then find the amplitude of $g(x)$ and sketch two periods of both functions on the same coordinate axes.

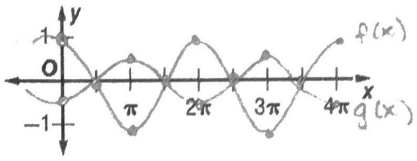
1. $f(x) = \sin x$

$g(x) = \frac{1}{3}\sin x$ amp = $\frac{1}{3}$

2. $f(x) = \cos x$

$g(x) = -\frac{1}{4}\cos x$ amp = $\frac{1}{4}$



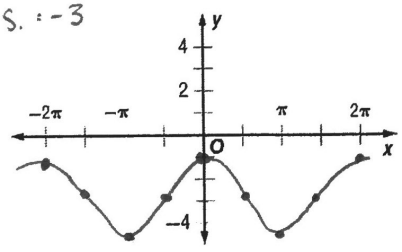


State the amplitude, period, frequency, phase shift, and vertical shift of each function. Then graph two periods of the function.

3. $y = 2\sin\left(x + \frac{\pi}{2}\right) - 3$

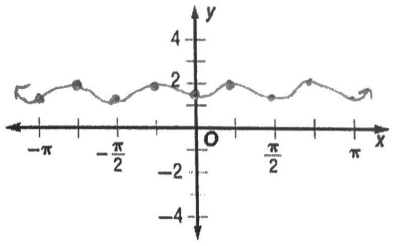
amp = 2
per = 2π
freq = $\frac{1}{2\pi}$
P.S = $\frac{\pi}{2}$ left
V.S. = −3

x	y
-2π	−1
$-\frac{3\pi}{2}$	−3
$-\pi$	−5
$-\frac{\pi}{2}$	−3
0	−1
$\frac{\pi}{2}$	−3



4. $y = \frac{1}{2}\cos(2x - \pi) + 2$

x	y
$-\pi$	1.5
$-\frac{3\pi}{4}$	2
$-\frac{\pi}{2}$	1.5
$-\frac{\pi}{4}$	2
0	1.5
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	



Write a sinusoidal function with the given amplitude, period, phase shift, and vertical shift.

5. sine function: amplitude = 15, period = 4π, phase shift = $\frac{\pi}{2}$, vertical shift = −10

A $4\pi = \frac{2\pi}{B}$ $\frac{\pi}{2} = -\frac{C}{\frac{1}{2}}$ D

$B = \frac{1}{2}$ $\frac{\pi}{2} = -2C$

$-4C = \pi$

$C = -\frac{\pi}{4}$

$y = 15\sin\left(\frac{1}{2}x - \frac{\pi}{4}\right) - 10$

6. cosine function: amplitude = $\frac{2}{3}$, period = $\frac{\pi}{3}$, phase shift = $-\frac{\pi}{3}$, vertical shift = 5

$\frac{\pi}{3} = \frac{2\pi}{B}$ $-\frac{\pi}{3} = -\frac{C}{6}$

$B\pi = 6\pi$ $-3C = -6\pi$

$B = 6$ $C = 2\pi$

$y = \frac{2}{3}\cos(6x - 2\pi) + 5$

7. MUSIC A piano tuner strikes a tuning fork note A above middle C and sets in motion vibrations that can be modeled by $y = 0.001\sin 880\pi$. Find the amplitude and period of the function.

amp = 0.001

per = $\frac{2\pi}{B}$

per = $\frac{2\pi}{880\pi}$

per = $\frac{1}{440}$

NAME _____ DATE _____ PERIOD _____

4-5 Practice

Graphing Other Trigonometric Functions

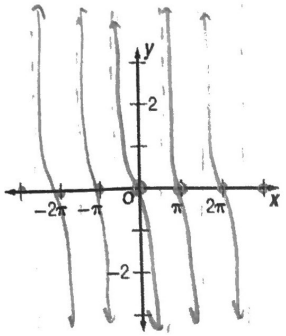
Locate the vertical asymptotes, and sketch the graph of each function.

phase shift = $\frac{-C}{B}$

$= \frac{-\frac{\pi}{3}}{2} = \left(-\frac{\pi}{6}\right)$ left

1. $y = -3 \tan x$

x	y
0	0
π	0

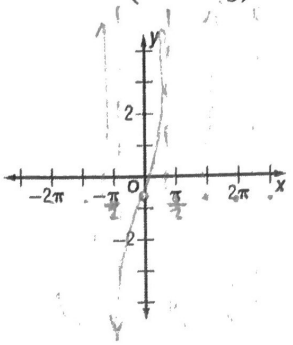


2. $y = -2 \cot\left(2x + \frac{\pi}{3}\right)$

cot asymptotes at $x = \pi n$

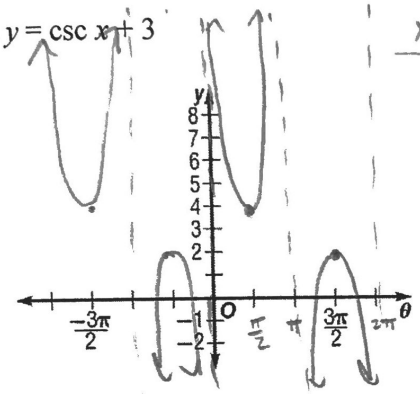
cot 2x asymptotes at $x = \frac{\pi}{2} n$

cot $(2x + \frac{\pi}{3})$ asymptotes at $x = \frac{\pi}{2} - \frac{\pi}{6}$



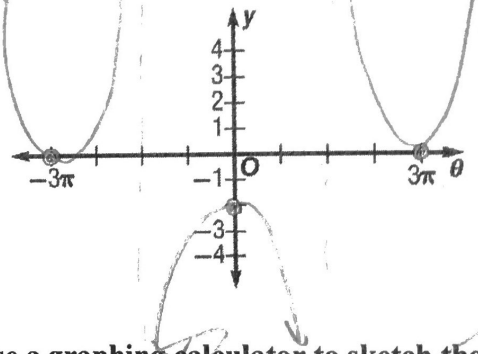
3. $y = \csc x + 3$

x	y
0	und
$\frac{\pi}{2}$	1+3=4
π	und
$\frac{3\pi}{2}$	2



4. $y = \sec\left(\frac{x}{3} + \pi\right) - 1$

x	y
0	−2
3π	0
-3π	0



Identify the damping factor $f(x)$ of the function. Then use a graphing calculator to sketch the graphs of $f(x)$, $-f(x)$, and the given function in the same viewing window. Describe the behavior of the graph.

5. $y = \frac{1}{2}x \cos 2x$

6. $y = -\frac{3}{2}x \sin \frac{\pi x}{2}$

7. MUSIC A guitar string is plucked at a distance of 0.6 centimeter above its resting position and then released, causing vibration. The damping constant of the guitar string is 1.8, and the note produced has a frequency of 105 cycles per second.

a. Write a trigonometric function that models the motion of the string.

b. Determine the amount of time t that it takes the string to be damped so that $-0.24 \leq y \leq 0.24$.

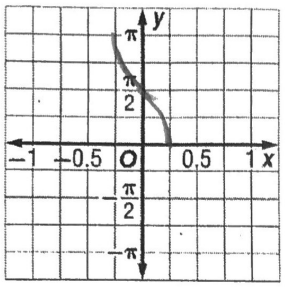
NAME _____ DATE _____ PERIOD _____

4-6 Practice

Inverse Trigonometric Functions

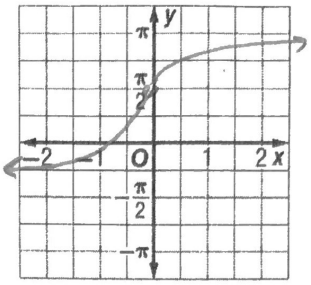
Sketch the graph of each function.

1. $y = \arccos 3x$

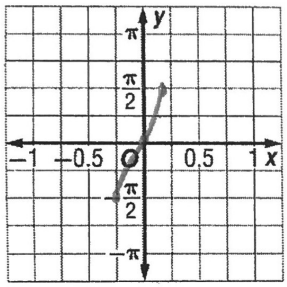


x	
0	$\cos^{-1}(0) = \frac{\pi}{2}$
$\frac{1}{4}$	$\cos^{-1}(\frac{3}{4}) = 0.722$
$\frac{1}{2}$	$\cos^{-1}(\frac{3}{2}) =$ und.

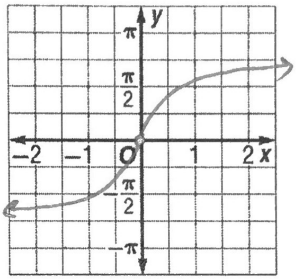
2. $y = \arctan x + 1$



3. $y = \sin^{-1} 3x$



4. $y = \tan^{-1} 3x$



Find the exact value of each expression, if it exists.

5. $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{6}$

6. $\cos^{-1}\left(\cos\frac{\pi}{3}\right) = \frac{\pi}{3}$

7. $\tan\left(-\frac{3\pi}{2}\right) =$ DNE (not between $-\frac{\pi}{2}$ & $\frac{\pi}{2}$)

8. $\sin^{-1}\left(\cos\frac{\pi}{3}\right) = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

9. $\arctan\left(-\frac{\sqrt{3}}{3}\right) = \tan^{-1}\left(-\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) = -\frac{\pi}{6}$ (sin / cos)

10. $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{3}$

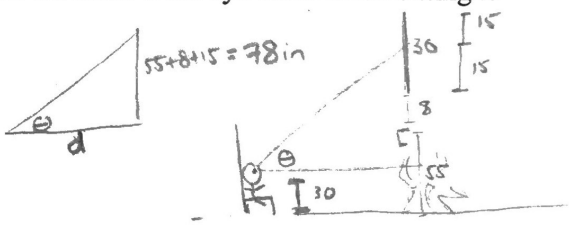
11. $\tan\left(\sin^{-1} 1 - \cos^{-1}\frac{1}{2}\right) = \frac{\sqrt{3}}{3}$
$\tan\left(\frac{\pi}{2} - \frac{\pi}{3}\right) = \tan\left(\frac{\pi}{6}\right) = \frac{\frac{1}{2}}{\sqrt{3}/2}$

12. $\sin\left(\arctan -\frac{\sqrt{3}}{3}\right) = -\frac{1}{2}$
$\sin\left(-\frac{\pi}{6}\right)$

13. **ART** Hans purchased a painting that is 30 inches tall that will hang 8 inches above the fireplace. The top of the fireplace is 55 inches from the floor.

a. Write a function modeling the maximum viewing angle θ for the distance d for Hans if his eye-level when sitting is 2.5 feet above the ground. $\cot\theta = \frac{d}{78}$

b. Determine the distance that corresponds to the maximum viewing angle. $78\cot\theta = d$



Write each trigonometric expression as an algebraic expression of x.

14. $\sin(\arccos x)$

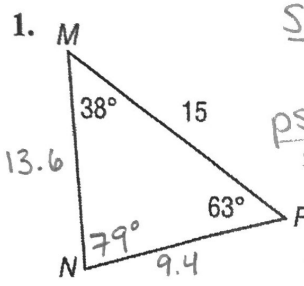
15. $\tan(\sin^{-1} x)$

NAME _____ DATE _____ PERIOD _____

4-7 Practice

The Law of Sines and the Law of Cosines

Solve each triangle. Round to the nearest tenth if necessary.

1. 

$\frac{\sin 79}{15} = \frac{\sin 63}{p}$

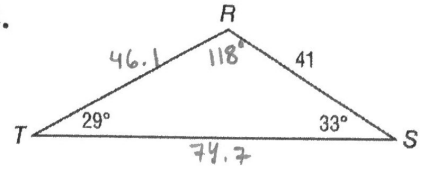
$\frac{p \sin 79}{\sin 79} = \frac{15 \sin 63}{\sin 79}$

$p = 13.6$

$\frac{\sin 79}{15} = \frac{\sin 38}{m}$

$\frac{m \sin 79}{\sin 79} = \frac{15 \sin 38}{\sin 79}$

$m = 9.4$

2. 

$\frac{\sin 29}{41} = \frac{\sin 118}{r}$

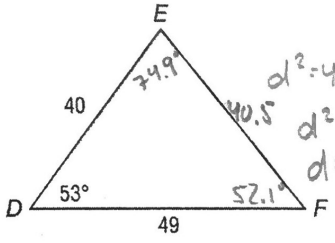
$\frac{r \sin 29}{\sin 29} = \frac{41 \sin 118}{\sin 29}$

$r = 74.7$

$\frac{\sin 29}{41} = \frac{\sin 33}{s}$

$\frac{s \sin 29}{\sin 29} = \frac{41 \sin 33}{\sin 29}$

$s = 46.1$

3. 

$d^2 = 40^2 + 49^2 - 2(40)(49)\cos(53)$

$d^2 = 1641.9$

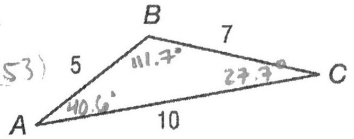
$d = 40.5$

$\frac{\sin 53}{40.5} = \frac{\sin F}{40}$

$\frac{40 \sin 53}{40.5} = \frac{40.5 \sin F}{40.5}$

$\sin F = 0.788$

$F = 52.1°$

4. 

$\frac{\sin 27.7}{5} = \frac{\sin A}{7}$

$\frac{7 \sin 27.7}{5} = \frac{5 \sin A}{5}$

$\sin A = 0.65$

$A = 40.6°$

$5^2 = 7^2 + 10^2 - 2(7)(10)\cos C$

$25 = 149 - 140\cos C$

$-124 = -140\cos C$

$0.8857 = \cos C$

$27.7° = C$

5. **STREET LIGHTING** A lamp post tilts toward the Sun at a 2° angle from the vertical and casts a 25-foot shadow. The angle from the tip of the shadow to the top of the lamp post is 45°. Find the length of the lamp post.

$\tan 45 = \frac{x}{25}$

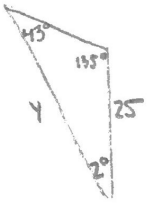
$x = 25 \tan 45$

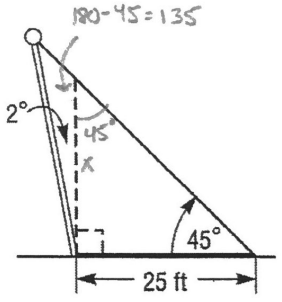
$x = 25$

$\frac{\sin 43}{25} = \frac{\sin 135}{y}$

$\frac{y \sin 43}{\sin 43} = \frac{25 \sin 135}{\sin 43}$

$y = 8.25$ ft





Use Heron's Formula to find the area of each triangle. Round to the nearest tenth.

6. $\triangle ABC$ if $a = 5$ ft, $b = 12$ ft, $c = 13$ ft

7. $\triangle FGH$ if $f = 11$ in., $g = 13$ in., $h = 16$ in.

8. $\triangle MNP$ if $m = 8$ yd, $n = 3.6$ yd, $p = 5.2$ yd

9. $\triangle XYZ$ if $x = 12$ cm, $y = 10$ cm, $z = 15.8$ cm

Find the area of each triangle to the nearest tenth.

10. $\triangle RST$ if $R = 115°$, $s = 15$ yd, $t = 20$ yd

11. $\triangle MNP$ if $n = 4$ ft, $P = 69°$, $N = 37°$

12. $\triangle DEF$ if $d = 2$ ft, $E = 85°$, $F = 19°$

13. $\triangle JKL$ if $j = 68$ cm, $l = 110$ cm, $K = 42.5°$