NAME Key DATE ____________ PERIOD ______

2-1 Study Guide and Intervention

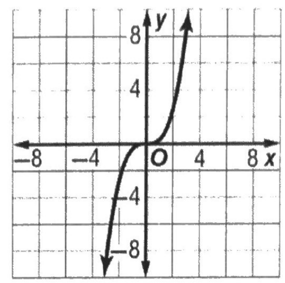
Power and Radical Functions

Power Functions A **power function** is any function of the form $f(x) = ax^n$ where a and n are nonzero constant real numbers. For example, $f(x) = 2x^2$, $f(x) = x^{\frac{1}{2}}$, or $f(x) = \sqrt{x}$ are power functions.

Example: Graph and analyze $f(x) = \frac{1}{3}x^3$. Describe the domain, range, intercepts, end behavior, continuity, and where the function is increasing or decreasing.

Evaluate the function for several x-values in its domain. Then use a smooth curve to connect each of these points to complete the graph.

x	−3	−2	−1	0	1	2	3
$f(x)$	−9	$-\frac{8}{3}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{8}{3}$	9



Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

Intercept: 0

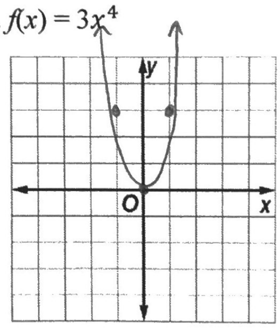
End behavior: $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$

Continuity: continuous for all real numbers Increasing: $(-\infty, \infty)$

Exercises

Graph and analyze each function. Describe the domain, range, intercepts, end behavior, continuity, and where the function is increasing or decreasing.

1. $f(x) = 3x^4$



x	y
−2	48
−1	3
0	0
1	3
2	48

domain: $(-\infty, \infty)$

range: $[0, \infty)$

x-y-int: $(0, 0)$

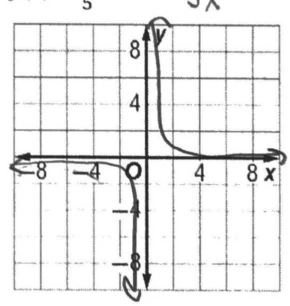
end behavior: $\lim_{x \to \pm\infty} f(x) = \infty$

continuity: $(-\infty, \infty)$

increasing: $(0, \infty)$

decreasing: $(-\infty, 0)$

2. $f(x) = \frac{1}{5}x^{-3} = \frac{1}{5x^3}$



x	y
−2	$-\frac{1}{40}$
2	$\frac{1}{40}$

domain: $x \neq 0, x \in \mathbb{R}$

range: $y \neq 0, y \in \mathbb{R}$

no intercepts

end behavior: $\lim_{x \to \pm\infty} f(x) = 0$

continuity: infinite discontinuity @ $x = 0$

increasing: $\emptyset$

decreasing: $(-\infty, 0) \cup (0, \infty)$

NAME ______________________ DATE ______________ PERIOD ______

2-1 Study Guide and Intervention *(continued)*

Power and Radical Functions

Radical Functions and Equations A **radical function** is a function that has at least one radical expression containing the independent variable. For example, $f(x) = 2\sqrt[4]{x^3}$ is a radical function.

A **radical equation** is an equation in which the variable is in the radicand. To solve a radical equation, isolate the radical expression. Then raise each side of the equation to the power of the index of the radical. Then check for **extraneous solutions.** These are solutions that do not satisfy the original equation.

Example: Solve $3 = \sqrt[3]{x^2 - 2x + 1} - 1$.

Step 1

$3 = \sqrt[3]{x^2 - 2x + 1} - 1$.	Original equation
$4 = \sqrt[3]{x^2 - 2x + 1}$	Isolate the radical.
$64 = x^2 - 2x + 1$	Cube each side.
$0 = x^2 - 2x - 63$	Subtract 64 from each side.
$0 = (x - 9)(x + 7)$	Factor.
$x - 9 = 0$ or $x + 7 = 0$	Zero Product Property
$x = 9$ $\quad x = -7$	Solve.

Step 2

Check both solutions.

$3 = \sqrt[3]{x^2 - 2x + 1} - 1$.	$3 = \sqrt[3]{x^2 - 2x + 1} - 1$.
$3 \stackrel{?}{=} \sqrt[3]{(9)^2 - 2(9) + 1} - 1$	$3 \stackrel{?}{=} \sqrt[3]{(-7)^2 - 2(-7) + 1} - 1$
$3 \stackrel{?}{=} \sqrt[3]{64} - 1$	$3 \stackrel{?}{=} \sqrt[3]{64} - 1$
$3 \stackrel{?}{=} 4 - 1$	$3 \stackrel{?}{=} 4 - 1$
$3 = 3$ ✓	$3 = 3$ ✓

Both solutions check, so the solutions are −7 and 9.

Exercises

Solve each equation.

1. $\sqrt[3]{x^2 - 1} - 6 = -4$

$\left(\sqrt[3]{x^2-1}\right)^3 = (2)^3$

$x^2 - 1 = 8$

$x^2 = 9$

$x = \pm 3$

2. $\left(\sqrt{6n - 3}\right)^2 = \left(\sqrt{-15 + 7n}\right)^2$

$6n - 3 = -15 + 7n$

$12 = n$

3. $4x = 21 + \sqrt{56 - x}$

$(4x - 21)^2 = \left(\sqrt{56 - x}\right)^2$

$16x^2 - 168x + 441 = 56 - x$

$16x^2 - 167x + 385 = 0$

$x = \dfrac{167 \pm \sqrt{(-167)^2 - 4(16)(385)}}{2(16)}$

$x = 7,\ x = \dfrac{55}{16}$

4. $\sqrt[5]{40 - 4x} + 15 = 17$

$\left(\sqrt[5]{40 - 4x}\right)^5 = (2)^5$

$40 - 4x = 32$

$4x = 8$

$x = 2$

NAME ______________________ DATE ______________________ PERIOD __________

2-2 Study Guide and Intervention
Polynomial Functions

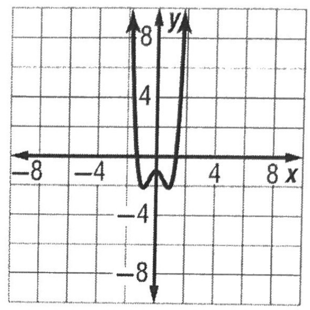
Graph Polynomial Functions A **polynomial function of degree *n*** is a function of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0,$$

where n is a nonnegative integer and $a_0, a_1, a_2, \ldots, a_{n-1}, a_n$ are real numbers with $a_n \neq 0$. Maxima and minima are located at **turning points**. A polynomial function of degree n has at most n distinct real zeros and at most $n - 1$ turning points.

Example 1: Graph $f(x) = 2x^4 - 3x^2 - 1$. Describe the end behavior of the graph of the polynomial function using limits. Explain your reasoning using the leading term test.

The degree is 4 and the leading coefficient is 2. Because the degree is even and the leading coefficient is positive, $\lim_{x\to-\infty} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = \infty$.



Example 2: State the number of possible real zeros and turning points of $f(x) = x^3 + 2x^2 - x - 2$. Then determine all of the real zeros by factoring.

The degree is 3, so f has at most 3 distinct real zeros and at most 3 – 1 or 2 turning points. To find the real zeros, solve the related equation $f(x) = 0$ by factoring.

$x^3 + 2x^2 - x - 2 = 0$	Set $f(x)$ equal to 0.
$x^2(x + 2) - 1(x + 2) = 0$	Group the terms and find the GCF.
$(x^2 - 1)(x + 2) = 0$	Regroup using the Distributive Property.
$(x + 1)(x - 1)(x + 2) = 0$	Factor completely.

So, f has three distinct real zeros, $x = -1$, $x = 1$, and $x = -2$. The graph has two turning points.

Exercises

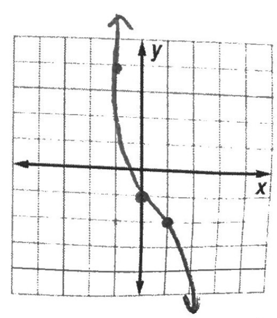
1. Graph the function $f(x) = -3x^5 + 2x^2 - 1$. Describe the end behavior of the graph of the polynomial function using limits. Explain your reasoning using the leading term test.

$\lim_{x\to-\infty} f(x) = \infty$

$\lim_{x\to+\infty} f(x) = -\infty$

Leading term = −
degree = odd

looks like



x	y
-2	
-1	4
0	-1
1	-2
2	

2. State the number of possible real zeros and turning points of
$f(x) = x^4 - 4x^3 + 5x^2 - 4x + 4$. Then determine all of the real zeros by factoring.

$\boxed{\text{possible zeroes: 4} \quad \text{turning points: 3}}$

$0 = x^4 - 4x^3 + 5x^2 - 4x + 4$

$\to (x-2)(x^3 - 2x^2 + x - 2)$

$(x-2)(x-2)(x^2+1)$

2 | 1 −4 5 −4 4
↓ 2 −4 2 −4
1 −2 1 −2 0

2 | 1 −2 1 −2
↓ 2 0 2
1 0 1 0

$\boxed{\text{real zeros: } x = 2}$

2-2 Study Guide and Intervention *(continued)*

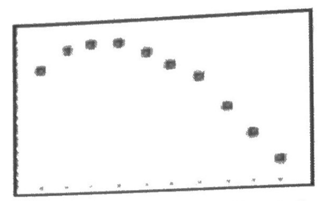
Polynomial Functions

Model Real-World Data with Polynomial Functions You can use a graphing calculator to model data by first creating a scatter plot.

Example: An oil tanker collides with another ship and starts leaking oil at the rate shown in the table. Write a polynomial function to model the set of data.

Time (h)	1	2	3	4	5	6	7	8	9	10
Flow rate (100s of L/h)	18.0	20.5	21.3	21.1	19.9	17.8	15.9	11.3	7.6	3.7

a. Create a scatter plot of the data and determine the type of polynomial function that could be used to represent the data.
Enter the data using the list feature. Let L1 be time and L2 flow rate.
The curve resembles a parabola, so a quadratic function can model the data.



[0, 11] scl: 1 by [0, 20] scl: 1

b. Write a polynomial function to model the data set. Round each coefficient to the nearest thousandth. State the correlation coefficient.
Use the **QUADREG** tool in the **STAT** menu to obtain $f(x) = -0.409x2 + 2.762x + 16.267$.
The correlation coefficient is about 0.99.

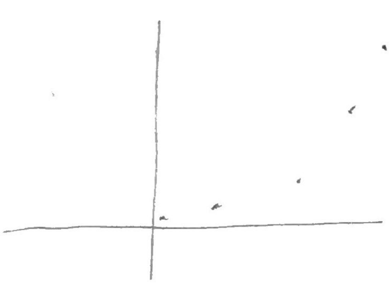
c. Use the model to estimate the flow rate in 10.5 hours.
Graph the unrounded regression as **Y1**. Press 2nd **[CALC]**, choose **VALUE** and enter 10.5 for x.
Since $y \approx 0.17$, the flow rate will be about 17 L/h.

d. Use the model to determine the approximate time the flow rate was 1000 liters per hour.
Graph the line $y = 10$. Press 2nd **[CALC]**, choose **intersect**, and choose each graph.
The intersection occurs when $x \approx 8.5$, so the time was about 8.5 hours.

Exercise

The farther a planet is from the Sun, the longer it takes to complete an orbit. Use a graphing calculator to write a polynomial function to model the set of data. Round each coefficient to the nearest thousandth. State the correlation coefficient.

Distance (AU)	0.39	0.72	1	1.49	5.19	9.51	19.1	30	39.3
Period (days)	88	225	365	687	4344	10,775	30,681	60,267	90,582



quadratic shape

quadreg $r^2 = 0.9997$ ← correlation coefficient (Catalog → Diagnostic On)

$\boxed{y = 34.968x^2 + 962.084x - 790.640}$

ME ______________________________ DATE ______________________ PERIOD __________

2-5 Study Guide and Intervention
Rational Functions

Graphs of Rational Functions Asymptotes are lines that a graph approaches.
For $f(x) = \frac{a(x)}{b(x)}$, where $a(x)$ and $b(x)$ have no common factors other than one, and n is the degree of $a(x)$ and m is the degree of $b(x)$:

- **Vertical asymptotes:** May occur at the real zeros of $b(x)$.

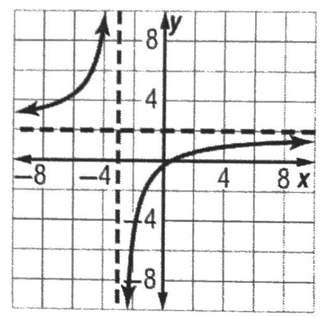
- **Horizontal asymptotes:**
  - If $n < m$, the asymptote is $y = 0$.
  - If $n = m$, the asymptote is $y = c$, where c is the ratio of the leading coefficients of the numerator and denominator.
  - If $n > m$, there is no horizontal asymptote.

- **Oblique asymptotes:** If $n = m + 1$, the equation is the quotient polynomial $q(x)$ of $f(x)$, or $\frac{a(x)}{b(x)} = q(x) + \frac{r(x)}{b(x)}$.

- x-intercepts, if any, occur at the real zeros of $a(x)$. The y-intercept, if it exists, is the value of f when $x = 0$.

Example: Determine any asymptotes and intercepts for $f(x) = \frac{2x-1}{x+3}$. Then graph the function and state its domain.

Vertical asymptotes: The zero of the denominator is –3. The vertical asymptote is $x = -3$.



Horizontal asymptotes: The degree of the numerator equals the degree of the denominator, so the horizontal asymptote is $y = 2$.

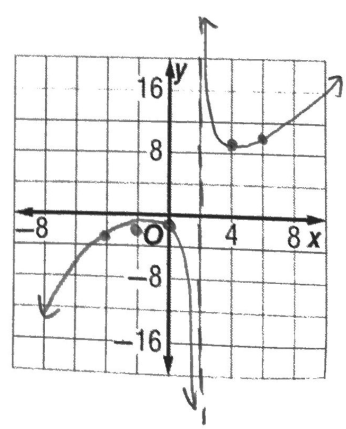
Intercepts: To find x-intercepts, find the zeros of the numerator by solving $2x - 1 = 0$. So, the x-intercept is $\frac{1}{2}$. To find the y-intercept, substitute 0 for x. The y-intercept is $-\frac{1}{3}$.
Graph the asymptotes and intercepts. Find and plot points in each interval.
$D = \{x \mid x \neq -3, x \in \mathbb{R}\}$

Exercise

Determine any asymptotes and intercepts for $f(x) = \frac{x^2+1}{x-2}$. Then graph the function and state its domain.

VA: x = 2
HA: none
y-int: (0, $-\frac{1}{2}$)
domain: x ≠ 2, x ∈ R

x	y
-4	17/-6 ≈ -3
-2	5/-4 ≈ -1
4	17/2 = 8.5
6	37/4 ≈ 9



2-5 Study Guide and Intervention *(continued)*

Rational Functions

Rational Equations Rational equations involving fractions can be solved by multiplying each term in the equation by the least common denominator (LCD) of all the terms of the equation. Always check your solutions to a rational equation as some may be extraneous.

Example: Solve each equation.

a. $x - \frac{8}{x-4} = 11$

$x - \frac{8}{x-4} = 11$	Original equation
$x(x-4) - \frac{8}{x-4}(x-4) = 11(x-4)$	Multiply each term by the LCD, $x - 4$.
$x^2 - 4x - 8 = 11x - 44$	Distributive Property
$x^2 - 15x + 36 = 0$	Simplify.
$(x-12)(x-3) = 0$	Factor.
$x = 12$ or 3	Zero Product Property

b. $\frac{x+1}{3(x-2)} = \frac{5x}{6} + \frac{1}{x-2}$

$\frac{x+1}{3(x-2)} = \frac{5x}{6} + \frac{1}{x-2}$	Original equation
$6(x-2)\left[\frac{x+1}{3(x-2)}\right] = 6(x-2)\left(\frac{5x}{6} + \frac{1}{x-2}\right)$	Multiply each side by the LCD, $6(x - 2)$.
$2(x+1) = (x-2)(5x) + 6(1)$	Multiply.
$2x + 2 = 5x^2 - 10x + 6$	Simplify.
$5x^2 - 12x + 4 = 0$	Write in standard form.
$(5x-2)(x-2) = 0$	Factor.
$5x - 2 = 0 \quad x - 2 = 0$	Zero Product Property
$x = \frac{2}{5} \quad x = 2$	Simplify.

Since x cannot equal 2 because a zero denominator results, the only solution is $\frac{2}{5}$.

Exercises

Solve each equation.

1. $x + \frac{10}{x} = 7$

$x \cdot x + \frac{10}{x} \cdot x = 7 \cdot x$

$x^2 + 10 = 7x$

$x^2 - 7x + 10 = 0$

$(x-5)(x-2) = 0$

$\boxed{x = 5 \quad x = 2}$

2. $\frac{15}{m} - m + 8 = 10$

$m \cdot \frac{15}{m} - m \cdot m = 2 \cdot m$

$15 - m^2 = 2m$

$0 = m^2 + 2m - 15$

$(x+5)(x-3) = 0$

$\boxed{x = -5 \quad x = 3}$

3. $\frac{4}{b-3} + \frac{3}{b} = \frac{-2b}{b-3}$

$b \cdot \frac{4}{b-3} + \frac{3}{b} \cdot b - 3 = \frac{-2b}{b-3} \cdot b$

$4b + 3b - 9 = -2b^2$

$7b - 9 = -2b^2$

$2b^2 + 7b - 9 = 0$

$(2b+9)(b-1) = 0$

$\boxed{b = -\frac{9}{2} \quad b = 1}$

4. $\frac{3}{x+1} + \frac{7}{2-x} = \frac{-4}{x-1}$

$(2-x)(x-1) \quad (x+1)(x-1) \quad (x+1)(2-x)$

$3(2x-2-x^2+x) + 7(x^2-1) = -4(2x-x^2+2-x)$

$3(-x^2+3x-2) + 7x^2 - 7 = -4(-x^2+x+2)$

$-3x^2 + 9x - 6 + 7x^2 - 7 = 4x^2 - 4x - 8$

$\cancel{4x^2} + 9x - 13 = \cancel{4x^2} - 4x - 8$

$13x = 5$

$x = \frac{5}{13}$

5. $\frac{x+3}{x} + \frac{7x}{x+3} = \frac{23}{4}$

$4(x+3) \quad 4x \quad x(x+3)$

$4(x^2+6x+9) + 28x^2 = 23x^2 + 69x$

$4x^2 + 24x + 36 + 28x^2 = 23x^2 + 69x$

$32x^2 + 24x + 36 = 23x^2 + 69x$

$\frac{9x^2 - 45x + 36 = 0}{9 \qquad\qquad\qquad 9}$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$\boxed{x = 4 \quad x = 1}$

6. $\frac{60}{x} - \frac{60}{x+3} = 1$

$\frac{60}{x} \cdot (x+3) - \frac{60}{x+3} \cdot x = 1 \cdot \frac{x(x+3)}{x(x+3)}$

$60x + 180 - 60x = x^2 + 3x$

$180 = x^2 + 3x$

$0 = x^2 + 3x - 180$

$0 = (x+15)(x-12)$

$\boxed{x = -15 \quad x = 12}$

NAME _______________ DATE _______________ PERIOD _______

2-6 Study Guide and Intervention
Nonlinear Inequalities

Polynomial Inequalities Real zeros divide the x-axis into intervals for which the value of $f(x)$ is positive (above the x-axis) or negative (below the x-axis). A **sign chart** shows these real zeros and the sign of $f(x)$ in that interval as either positive (+) or negative (–).

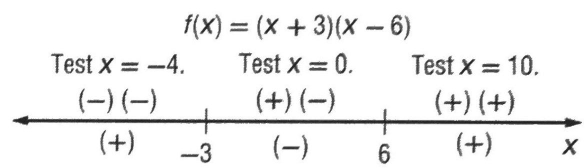
Example: Solve each inequality.

a. $x^2 – 3x – 15 > 3$

$x^2 - 3x - 15 > 3$	Original inequality
$x^2 - 3x - 18 > 0$	Subtract 3 from each side of the inequality.
$(x + 3)(x - 6) > 0$	Factor.

Let $f(x) = (x + 3)(x - 6)$.
$f(x)$ has real zeros at $x = -3$ and $x = 6$.

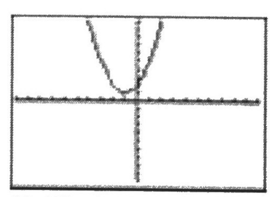
Create a sign chart using these real zeros. Substitute any number from each interval for x in the factored function rule. Write the sign of each factor and the sign of the product of the factors.



Because we are solving for when $x^2 - 3x - 18 > 0$, we choose the intervals for which the product is positive. The solution is $(-\infty, -3)$ or $(6, \infty)$.

b. $x^2 + 2x + 2 < 0$

The related function has no real zeros. The graph is always above the x-axis so the values of $f(x)$ are always positive. Because we are solving for when the values are negative, the equation has no real solution.



[–10, 10] scl: 1 by [–10, 10] scl: 1

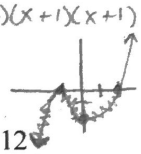
Exercises

Solve each inequality. $\boxed{(-\infty,-1)\cup(-1,2)}$

1. $x^3 - 3x - 2 < 0$

2| 1 0 -3 -2
  2 4 2
1 2 1 0

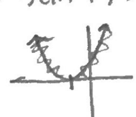
$(x-2)(x^2+2x+1)<0$
$(x-2)(x+1)(x+1)<0$



2. $x^2 + 2x + 1 > 0$

$(x+1)(x+1)>0$

$(-\infty,-1)\cup(-1,\infty)$

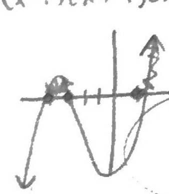


3. $x^3 + 6x^2 + 5x > 12$

$x^3+6x^2+5x-12>0$

1| 1 6 5 -12
  1 7 12
1 7 12 0

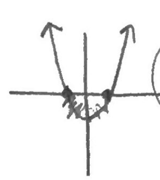
$(x-1)(x^2+7x+12)>0$
$(x-1)(x+4)(x+3)>0$



$(-4,-3)\cup(1,\infty)$

4. $3x^2 + x < 2$

$3x^2+x-2<0$
$(3x-2)(x+1)<0$



$(-1, \frac{2}{3})$

NAME ______________________ DATE ______________ PERIOD ________

2-6 Study Guide and Intervention *(continued)*

Nonlinear Inequalities

Rational Inequalities A **rational inequality** can change signs at its points of discontinuity as well as its real zeros. You must include the zeros of both the numerator and denominator in your sign chart.

Example: Solve $\frac{2}{x-4} + \frac{1}{x+1} > 0$.

$\frac{2}{x-4} + \frac{1}{x+1} > 0$ Original inequality

$\frac{2x+2+x-4}{(x-4)(x+1)} > 0$ Use the LCD, (x – 4) (x –1) , to combine the fractions.

$\frac{3x-2}{(x-4)(x+1)} > 0$ Simplify.

Let $f(x) = \frac{3x-2}{(x-4)(x+1)}$. The zeros and undefined points are the zeros of the numerator, $x = \frac{2}{3}$, and denominator, $x = 4$ and $x = -1$. Create a sign chart using these numbers. Test x-values in each interval.

Test $x = -5$.		Test $x = 0$.		Test $x = 1$.		Test $x = 10$.	
$\frac{(-)}{(-)(-)}$	undef.	$\frac{(-)}{(-)(+)}$	zero	$\frac{(+)}{(-)(+)}$	undef.	$\frac{(+)}{(+)(+)}$	
$(-)$	-1	$(+)$	$\frac{2}{3}$	$(-)$	4	$(+)$	x

The solutions are the intervals for which the final sign is positive: $(-1, \frac{2}{3})$ or $(4, \infty)$.

Exercises

Solve each inequality.

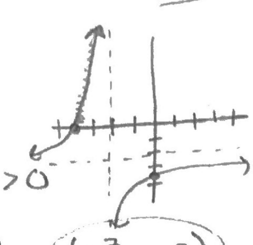
VA: $x=-2$
HA: $y=-2$
x-int: $(-\frac{7}{2}, 0)$

1. $\frac{x-1}{x+2} > 3$

$\frac{x-1}{x+2} - 3 > 0$

$\frac{x-1-3x-6}{x+2} > 0$

$\frac{-2x-7}{x+2} > 0$



$(-\frac{7}{2}, -2)$

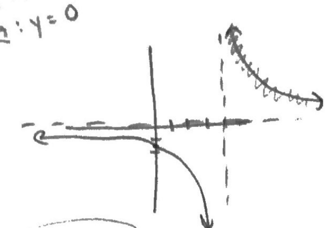
2. $\frac{x+2}{x-4} \le 1$

VA: $x=4$
HA: $y=0$

$\frac{x+2}{x-4} - 1 \le 0$

$\frac{x+2-x+4}{x-4} \le 0$

$\frac{6}{x-4} \le 0$



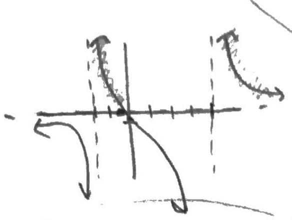
$(4, \infty)$

3. $\frac{1}{x+2} + \frac{3}{x-4} > 0$

$\frac{x-4+3x+6}{(x+2)(x-4)} > 0$

$\frac{4x+2}{(x+2)(x-4)} > 0$

$\frac{2(2x+1)}{(x+2)(x-4)} > 0$



$(-2, -\frac{1}{2}) \cup (4, \infty)$

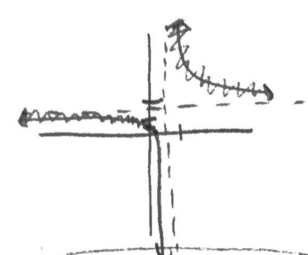
x-int: $(-\frac{1}{2}, 0)$
HA: $y=0$
VA: $x=-2, x=4$

4. $\frac{2x+1}{3x-2} \ge -1$

$\frac{2x+1}{3x-2} + 1 \ge 0$

$\frac{2x+1+3x-2}{3x-2} \ge 0$

$\frac{5x-1}{3x-2} \ge 0$



$(-\infty, \frac{1}{5}) \cup (\frac{2}{3}, \infty)$

x-int: $(\frac{1}{5}, 0)$
HA: $y = \frac{5}{3}$
VA: $x = \frac{2}{3}$

NAME ______________________ DATE ______________________ PERIOD ________

12-1 Study Guide and Intervention
Estimating Limits Graphically

Estimate Limits at Fixed Values

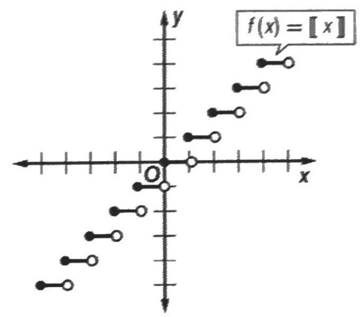
Left-Hand Limit	Right-Hand Limit
If the value of $f(x)$ approaches a unique number L_1 as x approaches c from the left, then $\lim_{x \to c^-} f(x) = L_1$.	If the value of $f(x)$ approaches a unique number L_2 as x approaches c from the right, then $\lim_{x \to c^+} f(x) = L_2$.

Existence of a Limit at a Point

The limit of a function $f(x)$ as x approaches c exists if and only if both one-sided limits exist and are equal. That is, if $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L$, then $\lim_{x \to c} f(x) = L$.

Example: Estimate each one-sided or two-sided limit, if it exists.

$\lim_{x \to 2^-} [\![x]\!]$, $\lim_{x \to 2^+} [\![x]\!]$, and $\lim_{x \to 2} [\![x]\!]$



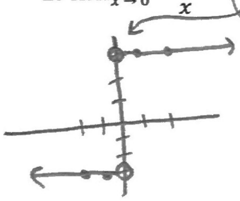
The graph of $f(x) = [\![x]\!]$ suggests that $\lim_{x \to 2^-} [\![x]\!] = 1$ and $\lim_{x \to 2^+} [\![x]\!] = 2$

Because the left- and right-hand limits of $f(x)$ as x approaches 2 are not the same, $\lim_{x \to 2} [\![x]\!]$ does not exist.

Exercises

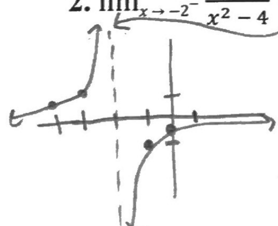
Estimate each one-sided or two-sided limit, if it exists.

1. $\lim_{x \to 0^+} \frac{|3x|}{x} = 3$



x	y
-2	-3
-1	-3
0	und.
1	3
2	3

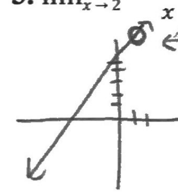
2. $\lim_{x \to -2^-} \frac{|x-2|}{x^2-4} = \infty$



x	y
-4	$\frac{1}{2}$
-3	1
-2	und.
-1	-1
0	$-\frac{1}{2}$
1	$-\frac{1}{3}$

3. $\lim_{x \to 2} \frac{x^2+3x-10}{x-2} = 7$

$\frac{(x+5)\cancel{(x-2)}}{\cancel{x-2}}$



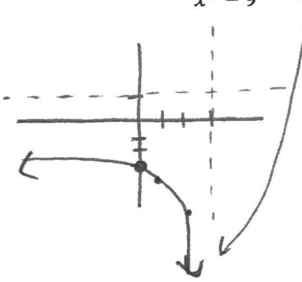
4. $\lim_{x \to 0}(1 - \cos^2 x) = 0$

$1-\cos^2(0)$ $\qquad \cos(0) = 1$

$1-(1)^2$

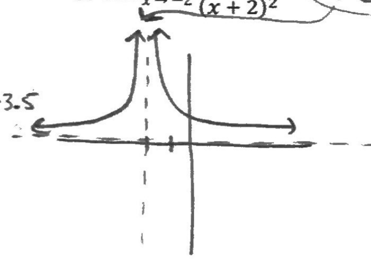
0

5. $\lim_{x \to 3^-} \frac{x^2+27}{x^2-9} = -\infty$



x	y
0	-3
1	$28/-8 \approx -3.5$
2	-6
3	und.

6. $\lim_{x \to -2} \frac{1}{(x+2)^2} = +\infty$



NAME ____________________ DATE ____________________ PERIOD ________

12-1 study Guide and Intervention *(continued)*

Estimating Limits Graphically

Estimate Limits at Infinity

- If the value of $f(x)$ approaches a unique number L_1 as x increases, then $\lim_{x \to \infty} f(x) = L_1$.
- If the value of $f(x)$ approaches a unique number L_2 as x decreases, then $\lim_{x \to -\infty} f(x) = L_2$.

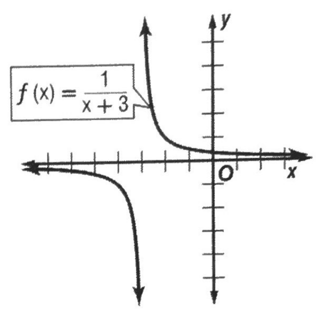
Estimate $\lim_{x \to \infty} \frac{1}{x+3}$, if it exists.

Analyze Graphically The graph of $f(x) = \frac{1}{x+3}$ suggests that $\lim_{x \to \infty} \frac{1}{x+3} = 0$.
As x increases, the height of the graph gets closer to 0. The limit indicates a horizontal asymptote at $y = 0$.

Support Numerically Make a table of values, choosing x-values that grow increasingly large.

x approaches infinity

x	10	100	1000	10,000	100,000
f(x)	0.08	0.01	0.01	0.0001	0.00001

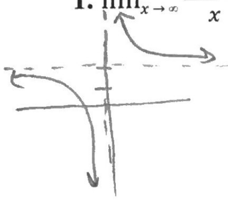


The pattern of outputs suggests that as x grows increasingly larger, $f(x)$ approaches 0. This supports our graphical analysis

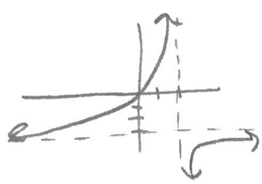
Exercises

Estimate each limit, if it exists.

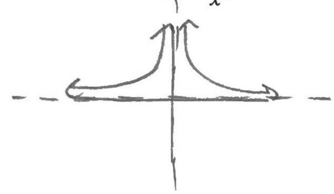
1. $\lim_{x \to \infty} \frac{2x+1}{x} = 2$



2. $\lim_{x \to -\infty} \frac{-3x+1}{x-2} = -3$

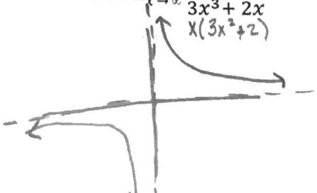


3. $\lim_{x \to \infty} \frac{1}{x^2} = 0$



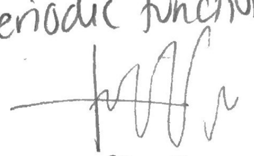
4. $\lim_{x \to \infty} \frac{2x+5}{3x^3+2x} = 0$

$x(3x^2+2)$

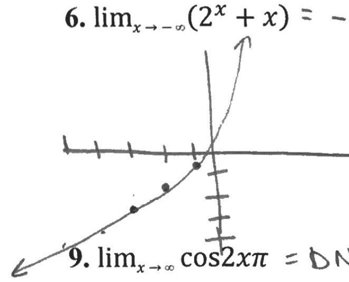


5. $\lim_{x \to \infty} (e^x \sin 2x\pi) =$ DNE

periodic function



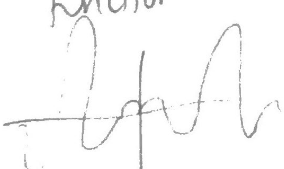
6. $\lim_{x \to -\infty} (2^x + x) = -\infty$



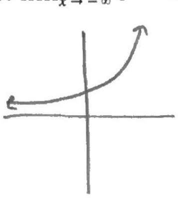
X	Y
-5	$\frac{1}{32} + -5 = -4\frac{31}{32}$
-4	$\frac{1}{16} + -4 = -3\frac{15}{16}$
-3	$\frac{1}{8} + -3 = -2\frac{7}{8}$
-2	$\frac{1}{4} + -2 = -1\frac{3}{4}$
-1	$\frac{1}{2} + -1 = -\frac{1}{2}$

7. $\lim_{x \to \infty} (x \sin x) =$ DNE

periodic function

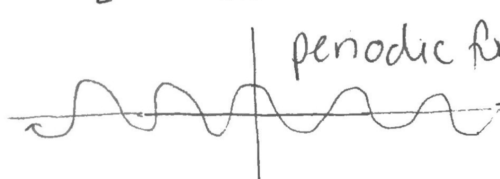


8. $\lim_{x \to -\infty} e^{2x} = 0$



9. $\lim_{x \to \infty} \cos 2x\pi =$ DNE

periodic function



* don't worry about periodic functions yet!
(# 5, 7, 9) . . . or # 6

12-2 Study Guide and Intervention

Evaluating Limits Algebraically

Compute Limits at a Point

Example 1: Use direct substitution, if possible, to evaluate $\lim_{x \to -2}(-2x^4 + 3x^3 - 5x + 3)$.

Since this is the limit of a polynomial function, we can apply the method of direct substitution to find the limit.

$\lim_{x \to -2}(-2x^4 + 3x^3 - 5x + 3) = -2(-2)^4 + 3(-2)^3 - 5(-2) + 3$

$= -32 - 24 + 10 + 3$, or -43

Example 2: Use factoring to evaluate $\lim_{x \to 4} \frac{x^2 - 9x + 20}{x - 4}$.

$\lim_{x \to 4} \frac{x^2 - 9x + 20}{x - 4} = \lim_{x \to 4} \frac{(x-5)(x-4)}{(x-4)}$	Factor.
$= \lim_{x \to 4}(x - 5)$	Divide out the common factor and simplify.
$= 4 - 5$, or -1	Apply direct substitution and simplify.

Example 3: Use rationalizing to evaluate $\lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16}$.

By direct substitution, you obtain $\frac{\sqrt{16} - 4}{16 - 16}$ or $\frac{0}{0}$. Rationalize the numerator of the fraction before factoring and dividing common factors.

$\lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16} = \lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16} \cdot \frac{\sqrt{x} + 4}{\sqrt{x} + 4}$	Multiply the numerator and denominator by $\sqrt{x} + 4$, the conjugate of $\sqrt{x} - 4$.
$= \lim_{x \to 16} \frac{x - 16}{(x - 16)(\sqrt{x} + 4)}$	Simplify.
$= \lim_{x \to 16} \frac{\cancel{x - 16}}{\cancel{(x - 16)}(\sqrt{x} + 4)}$	Divide out the common factor.
$= \lim_{x \to 16} \frac{1}{(\sqrt{x} + 4)}$	Simplify.
$= \frac{1}{\sqrt{16} + 4}$ or $\frac{1}{8}$	Apply direct substitution and simplify.

Exercises

Evaluate each limit.

1. $\lim_{x \to 3}(2x^2 - 5x)$ = 3

$2(3)^2 - 5(3)$
$18 - 15$

2. $\lim_{x \to 5} \sqrt{x^3 - 4}$ = 11

$\sqrt{5^3 - 4}$
$\sqrt{121}$

3. $\lim_{x \to -2} \frac{x^2 + 9x + 14}{x + 2}$ = 5

$(x+7)\cancel{(x+2)}$

hole $-2+7=5$

4. $\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$

$\frac{\cancel{x-4}}{\cancel{(x-4)}(\sqrt{x}+2)} = \frac{1}{\sqrt{x}+2}$

$= \frac{1}{\sqrt{(4)}+2}$

$= \frac{1}{4}$

5. $\lim_{x \to -4} \left(\frac{1}{x} + x\right) = -\frac{17}{4}$

$\frac{1}{-4} + (-4)$

$-4\frac{1}{4}$

6. $\lim_{x \to 2}(-x^2 + 5x - 1)$ = 5

$(-(2)^2 + 5(2) - 1)$
$(-4 + 10 - 1)$
5

12-2 Study Guide and Intervention *(continued)*
Evaluating Limits Algebraically

Compute Limits at Infinity

Limits of Power Functions at Infinity	Limits of Polynomials at Infinity	Limits of Reciprocal Functions at Infinity
For any positive integer n, • $\lim_{x\to\infty} x^n = \infty$. • $\lim_{x\to-\infty} x^n = \infty$ if n is even. • $\lim_{x\to-\infty} x^n = \infty$ if n is odd.	Let p be a polynomial function $p(x) = a_n x^n + \ldots + a_1 x + a_0$. Then $\lim_{x\to\infty} p(x) = \lim_{x\to\infty} a_n x^n$ and $\lim_{x\to-\infty} p(x) = \lim_{x\to-\infty} a_n x^n$.	For any positive integer n, $\lim_{x\to\pm\infty} \frac{1}{x^n} = 0$.

Example : Evaluate each limit.

a. $\lim_{x\to-\infty}(x^5 - 6x + 1)$

$\lim_{x\to-\infty}(x^5 - 6x + 1) = \lim_{x\to-\infty} x^5$ Limits of Polynomials at Infinity

$= -\infty$ Limits of Power Functions at Infinity

b. $\lim_{x\to\infty}(2x^4 + 5x^2)$

$\lim_{x\to\infty}(2x^4 + 5x^2) = \lim_{x\to\infty} 2x^4$ Limits of Polynomials at Infinity

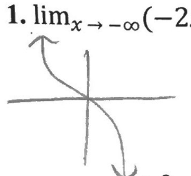
$= 2\lim_{x\to\infty} x^4$ Scalar Multiple Property

$= 2 \cdot \infty = \infty$ Limits of Power Functions at Infinity

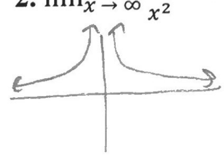
Exercises

Evaluate each limit.

1. $\lim_{x\to-\infty}(-2x^3 + 5x)$ $= \infty$



2. $\lim_{x\to\infty}\frac{5}{x^2}$ $= 0$



(HA)

3. $\lim_{x\to\infty}\frac{6x-1}{10x+7}$ $= \frac{6}{10} = \frac{3}{5}$

(HA)

4. $\lim_{x\to\infty}\frac{6x^2-2x}{x^3+1}$ $= 0$

(HA)

5. $\lim_{x\to\infty}\frac{5x^4+2x^3-1}{2x^3+x^2-1}$ $= \infty$

(no HA)

6. $\lim_{x\to-\infty}(3x^3 + 5x - 1)$ $= -\infty$