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3-1 Study Guide and Intervention *(continued)*
Exponential Functions

Exponential Growth and Decay Many real-world situations can be modeled by exponential functions. One of the equations below may apply.

Exponential Growth or Decay	Continuous Exponential Growth or Decay	Compound Interest
$N = N_0(1 + r)^t$ N is the final amount, N_0 is the initial amount, r is the rate of growth or decay, and t is time.	$N = N_0 e^{kt}$ N is the final amount, N_0 is the initial amount, k is the rate of growth or decay, t is time, and e is a constant.	$A = P\left[1 + \frac{r}{n}\right]^{nt}$ P is the principal or initial investment, A is the final amount of the investment, r is the annual interest rate, n is the number of times interest is compounded each year, and t is the number of years.

Example 1: BIOLOGY A researcher estimates that the initial population of a colony of cells is 100. If the cells reproduce at a rate of 25% per week, what is the expected population of the colony in six weeks?

$N = N_0(1 + r)^t$ — Exponential Growth Formula

$= 100(1 + 0.25)^6$ — $N_0 = 100$, $r = 0.25$, $t = 6$

≈ 381.4697266 — Use a calculator.

There will be about 381 cells in the colony in 6 weeks.

Example 2: FINANCIAL LITERACY Lance has a bank account that will allow him to invest $1000 at a 5% interest rate compounded continuously. If there are no other deposits or withdrawals, what will Lance's account balance be after 10 years?

$A = Pe^{rt}$ — Continuous Compound Interest Formula

$= 1000e^{(0.05)(10)}$ — $P = 1000$, $r = 0.05$, and $t = 10$

≈ 1648.72 — Simplify.

With continuous compounding, Lance's account balance after 10 years will be $1648.72.

Exercises

1. **FINANCIAL LITERACY** Compare the balance after 10 years of a $5000 investment earning 8.5% interest compounded continuously to the same investment compounded quarterly.

$5000\left(1+\frac{0.085}{4}\right)^{4\cdot 10} = \$11{,}594.52$ $\quad$ $5000e^{0.085\cdot 10} = \$11{,}698.23$

2. **ENERGY** In 2007, it is estimated that the United States used about 101,000 quadrillion thermal units. If U.S. energy consumption increases at a rate of about 0.5% annually, what amount of energy will the United States use in 2020?

$101{,}000(1+0.005)^{13} = 107{,}765.6$ quadrillion tu

3. **BIOLOGY** The number of rabbits in a field showed an increase of 10% each month over the last year. If there were 10 rabbits at this time last year, how many rabbits are in the field now?

$10(1+0.10)^{12} = 31$ rabbits

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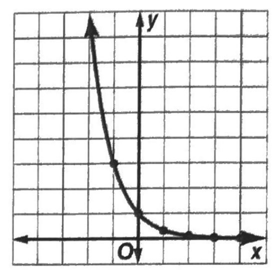
3-1 Study Guide and Intervention

Exponential Functions

Exponential Functions An **exponential function** with base b has the form $f(x) = ab^2$, where x is any real number and a and b are real number constants such that $a \neq 0$, b is positive, and $b \neq 1$. If $b > 1$, then the function is *exponential growth.* If $0 < b < 1$, then the function is *exponential decay*.

Example: Sketch and analyze the graph of $f(x) = \left(\frac{1}{3}\right)^x$. Describe its domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

x	–3	–2	–1	0	1	2	3
$f(x)$	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$



Domain: $(-\infty, \infty)$ Range: $(0, \infty)$

Intercept: $(0, 1)$ Asymptote: x-axis

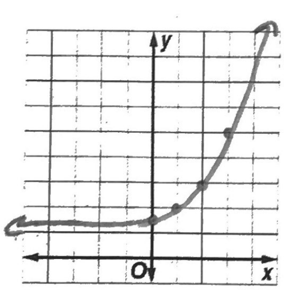
End behavior: $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = 0$

Decreasing: $(-\infty, \infty)$

Exercises

Sketch and analyze the graph of each function. Describe its domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

1. $h(x) = 2^{x-1} + 1$



x	y
0	$1\frac{1}{2}$
1	2
2	3
3	5

domain: $\mathbb{R}$

range: $y > 1$

intercept: $(0, \frac{3}{2})$

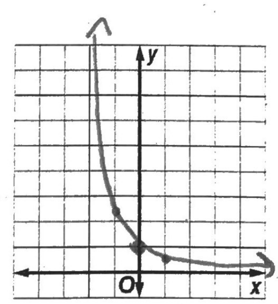
asymptote: $y = 1$

end beh: $\lim_{x \to -\infty} h(x) = 1$

$\lim_{x \to \infty} h(x) = \infty$

increasing: $(-\infty, \infty)$

2. $k(x) = e^{-2x}$



x	y
0	1
$\frac{1}{2}$	$\frac{1}{e}$
$-\frac{1}{2}$	e

domain: $\mathbb{R}$

range: $y > 0$

intercept: $(0, 1)$

asymptote: $y = 0$

end behavior: $\lim_{x \to -\infty} k(x) = \infty$

$\lim_{x \to \infty} k(x) = 0$

decreasing: $(-\infty, \infty)$

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3-2 Study Guide and Intervention *(continued)*
Logarithmic Functions

Graphs of Logarithmic Functions The inverse of $f(x) = b^x$ is called the logarithmic function with base b, or $f(x) = \log_b x$, and read *f of x equals the log base b of x.*

Example: Sketch and analyze the graph of $f(x) = \log_6 x$. Describe its domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

Create a table of values for the inverse of the function, the exponential function $f^{-1}(x) = 6^x$.

x	−2	−1	0	1	2
$f^{-1}(x)$	0.028	0.17	1	6	36

Since the functions are inverses, you can obtain the graph of $f(x)$ by plotting the points $(f^{-1}(x), x)$.

Domain: $(0, \infty)$

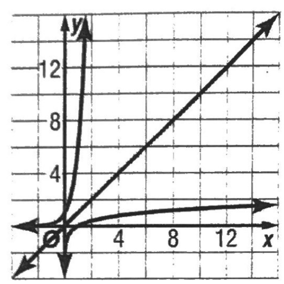
Range: $(-\infty, \infty)$

x-intercept: $(1, 0)$

Asymptote: y-axis

End behavior: $\lim_{x \to 0^+} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$

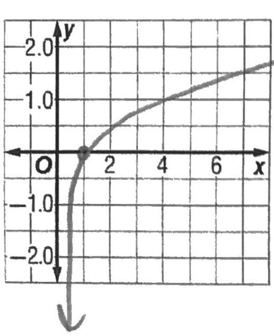
Increasing: $(0, \infty)$



Exercises

Sketch and analyze the graph of each function below. Describe its domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

1. $g(x) = \log_3 x$



x	y
1	0
9	2

domain: $x > 0$

range: $\mathbb{R}$

x-int: $(1, 0)$

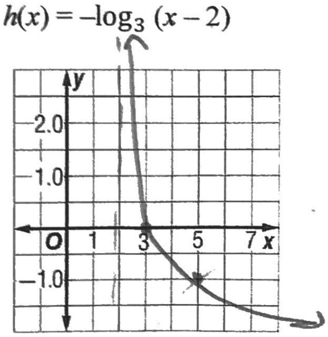
asymptote: $x = 0$

end behavior: $\lim_{x \to 0^+} g(x) = -\infty$

$\lim_{x \to \infty} g(x) = \infty$

increasing: $(0, \infty)$

2. $h(x) = -\log_3 (x - 2)$



x	y
5	−1
3	0

domain: $x > 2$

range: $\mathbb{R}$

x-int: $0 = -\log_3(x-2)$

$0 = \log_3(x-2)$

$3^0 = x - 2$

$1 = x - 2$

$3 = x$

asymptote: $x = 2$

end behavior:

$\lim_{x \to 2^+} h(x) = \infty$

$\lim_{x \to \infty} h(x) = -\infty$

decreasing: $(2, \infty)$

3-2 Study Guide and Intervention
Logarithmic Functions

Logarithmic Functions and Expressions The inverse relationship between logarithmic functions and exponential functions can be used to evaluate logarithmic expressions.

If $b > 0$, $b \neq 1$, and $x > 0$, then

Logarithmic Form		**Exponential Form**
$\log_b x = y$	if and only if	$b^y = x$
base, exponent		base, exponent

The following properties are also useful.

$\log_b 1 = 0$ $\qquad$ $\log_b b = 1$ $\qquad$ $\log_b b^x = x$ $\qquad$ $b^{\log_b x} = x, x > 0$

Example 1: Evaluate each logarithm.

a. $\mathbf{\log_5 \frac{1}{25}}$

$\log_5 \frac{1}{25} = y$ — Let $\log_5 \frac{1}{25} = y$.

$5^y = \frac{1}{25}$ — Write in exponential form.

$5^y = 5^{-2}$ — $\frac{1}{25} = 5^{-2}$

$y = -2$ — Equality Prop. of Exponents

Therefore, $\log_5 \frac{1}{25} = -2$ because $5^{-2} = \frac{1}{25}$.

b. $\mathbf{\log_3 \sqrt{3}}$

$\log_3 \sqrt{3} = y$ — Let $\log_3 \sqrt{3} = y$.

$3^y = \sqrt{3}$ — Write in exponential form.

$3^y = 3^{\frac{1}{2}}$ — $3^{\frac{1}{2}} = \sqrt{3}$

$y = \frac{1}{2}$ — Equality Prop. of Exponents

Therefore, $\log_3 \sqrt{3} = \frac{1}{2}$ because $3^{\frac{1}{2}} = \sqrt{3}$.

Example 2: Evaluate each expression.

a. $\mathbf{\ln e^7}$
$\ln e^7 = 7$ $\qquad$ $\ln e^x = x$

b. $\mathbf{e^{\ln 5}}$
$e^{\ln 5} = 5$ $\qquad$ $e^{\ln 5} = x$

c. $\mathbf{10^{\log 13}}$
$10^{\log 13} = 13$ $\qquad$ $10^{\log x} = x$

Exercises

Evaluate each logarithm.

1. $\log_7 7$ = 1

2. $10^{\log 5x}$ = 5x
$\log_{10} y = \log 5x$

3. $3^{\log_3 2}$ = 2
$\log_3 x = \log_3 2$

4. $\log_6 36$ = 2

5. $\log_3 \frac{1}{81}$ = −4

6. $e^{\ln x^2}$ = x^2
$\ln_e y = \ln x^2$

7. FINANCIAL LITERACY Ms. Dasilva has \$3000 to invest. She would like to invest in an account that compounds continuously at 6%. Use the formula $\ln A - \ln P = rt$, where A is the current balance, P is the original principal, r is the rate as a decimal, and t is the time in years. How long will it take for her balance to be \$6000?

$\ln 6000 - \ln 3000 = 0.06t$

$\ln \frac{6000}{3000} = 0.06t$

$\ln 2 = 0.06t$

$\frac{\ln(2)}{0.06} = t$

$t = 11.55$ years

3-3 Study Guide and Intervention *(continued)*

Properties of Logarithms

Change of Base Formula If the logarithm is in a base that needs to be changed to a different base, the **Change of Base Formula** is required.

For any positive real numbers a, b, and x, $a \neq 1$, $b \neq 1$, $\log_b x = \frac{\log_a x}{\log_a b}$.

Many non-graphing calculators cannot be used for logarithms that are not base e or base 10. Therefore, you will often use this formula, especially for scientific applications. Either of the following forms will provide the correct answer.

$$\log_b x = \frac{\log x}{\log b} \qquad \log_b x = \frac{\ln x}{\ln b}$$

Example: Evaluate each logarithm.

a. $\log_2 7$

$\log_2 7 = \frac{\ln 7}{\ln 2}$ Change of Base Formula

≈ 2.81 Use a calculator.

b. $\log_{\frac{1}{3}} 10$

$\log_{\frac{1}{3}} 10 = \frac{\log 10}{\log \frac{1}{3}}$ Change of Base Formula

≈ -2.10 Use a calculator.

Exercises

Evaluate each logarithm.

1. $\log_{32} 631$ ≈ 1.86

2. $\log_3 17$ ≈ 2.58

3. $\log_7 1094$ ≈ 3.60

4. $\log_6 94$ ≈ 2.54

5. $\log_5 256$ ≈ 3.45

6. $\log_9 712$ ≈ 2.99

7. $\log_6 832$ ≈ 3.75

8. $\log_{11} 47$ ≈ 1.61

9. $\log_3 9$ = 2

10. $\log_8 256$ ≈ 2.67

11. $\log_{12} 4302$ ≈ 3.37

12. $\log_{0.5} 420$ ≈ −8.71

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3-3 Study Guide and Intervention

Properties of Logarithms

Properties of Logarithms Since logarithms and exponents have an inverse relationship, they have certain properties that can be used to make them easier to simplify and solve.

If b, x, and y are positive real numbers, $b \neq 1$, and p is a real number, then the following statements are true.

- $\log_b xy = \log_b x + \log_b y$ Product Property
- $\log_b \frac{x}{y} = \log_b x - \log_b y$ Quotient Property
- $\log_b x^p = p \log_b x$ Power Property

Example 1: Evaluate $3 \log_2 8 + 5 \log_2 \frac{1}{2}$.

$3 \log_2 8 + 5 \log_2 \frac{1}{2} = 3 \log_2 2^3 + 5 \log_2 2^{-1}$	$8 = 2^3$; $2^{-1} = \frac{1}{2}$
$= 3(3 \log_2 2) + 5(-\log_2 2)$	Power Property
$= 3(3)(1) + 5(-1)(1)$	$\log_x x = 1$
$= 4$	Simplify.

Example 2: Expand $\ln \frac{8x^5}{3y^2}$.

$\ln \frac{8x^5}{3y^2} = \ln 8x^5 - \ln 3y^2$	Quotient Property
$= \ln 8 + \ln x^5 - \ln 3 - \ln y^2$	Product Property
$= \ln 8 + 5 \ln x - \ln 3 - 2 \ln y$	Power Property

Exercises

1. Evaluate $2 \log_3 27 + 4 \log_3 \frac{1}{3}$.

$2(3) + 4(-1)$

$6 - 4 = 2$

Expand each expression.

2. $\log_3 \frac{5r^5}{\sqrt[3]{t^2}} = \log_3 5r^5 - \log_3 \sqrt[3]{t^2}$

$\log_3 5 + \log_3 r^5 - \log_3 t^{2/3}$

$\boxed{\log_3 5 + 5\log_3 r - \frac{2}{3}\log_3 t}$

3. $\log \frac{(a-2)(b+4)^6}{9(b-2)^5} = \log (a-2)(b+4)^6 - \log 9(b-2)^5$

$\log(a-2) + \log(b+4)^6 - [\log 9 + \log(b-2)^5]$

$\log(a-2) + 6\log(b+4) - \log 9 - 5\log(b-2)$

Condense each expression.

4. $11 \log_9 (x-3) - 5 \log_9 2x$

$\log_9 (x-3)^{11} - \log_9 (2x)^5$

$\boxed{\log_9 \frac{(x-3)^{11}}{32x^5}}$

5. $\frac{3}{4} \ln (2h-k) + \frac{3}{5} \ln (2h+k)$

$\ln (2h-k)^{3/4} + \ln (2h+k)^{3/5}$

$\ln\left[\sqrt[4]{(2h-k)^3} \cdot \sqrt[5]{(2h+k)^3}\right]$

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3-4 Study Guide and Intervention

Exponential and Logarithmic Equations

Solve Exponential Equations **One-to-One Property of Exponential Functions:** For $b > 0$ and $b \neq 1$, $bx = by$ if and only if $x = y$. This property will help you solve exponential equations. For example, you can express both sides of the equation as an exponent with the same base. Then use the property to set the exponents equal to each other and solve. If the bases are not the same, you can *exponentiate* each side of an equation and use logarithms to solve the equation.

Example 1

a. Solve $4^{x-1} = 16^x$.

$4^{x-1} = 16^x$	Original equation
$4^{x-1} = (4^2)^x$	$16 = 4^2$
$4^{x-1} = 4^{2x}$	Power of a Power
$x - 1 = 2x$	One-to-One Property
$-1 = x$	Subtract x from each side.

b. Solve $e^{2x} - 3e^x + 2 = 0$.

$e^{2x} - 3e^x + 2 = 0$	Original equation
$u^2 - 3u + 2 = 0$	Write in quadratic form.
$(u-2)(u-1) = 0$	Factor.
$u = 2$ or $u = 1$	Solve.
$e^x = 2$ or $e^x = 1$	Substitute for u.
$x = \ln 2$ or 0	Take the natural logarithm of each side

Example 2: Solve each equation. Round to the nearest hundredth if necessary.

a. $3^x = 19$

$\log 3^x = \log 19$	Take the log of both sides.
$x \log 3 = \log 19$	Power Property
$x = \frac{\log 19}{\log 3}$	Divide each side by log 3.
$x \approx 2.68$	Use a calculator. Check this solution in the original equation.

b. $e^{8x+1} - 6 = 1$

$e^{8x+1} = 7$	Add 6 to both sides.
$\ln e^{8x+1} = \ln 7$	Take the ln of both sides.
$(8x + 1) \ln e = \ln 7$	Power Property
$8x + 1 = \ln 7$	$\ln e = 1$
$8x = \ln 7 - 1$	Subtract 1 from each side.
$x = \frac{\ln 7 - 1}{8} \approx 0.12$	Divide by 8 and use a calculator.

Exercises

Solve each equation. Round to the nearest hundredth.

1. $9^x = 3^{3x-4}$

X=4

2. $\left(\frac{1}{4}\right)^{2x-1} = \left(\frac{1}{8}\right)^{11-x}$

X=5

3. $4^{3x-2} = \frac{1}{2}^{2x}$

X= 1/2

4. $2e^{2x} + 12e^x - 54 = 0$

ln 3

5. $9^{2x} = 12$

X=0.57

6. $2.4e^{x-6} = 9.3$

X=7.35

7. $3^{2x} = 6^{x-1}$

X=-4.42

8. $e^{19x} = 23$

X=0.17

1. $9^{x} = 3^{3x-4}$

$(3^{2})^{x} = 3^{3x-4}$

$3^{2x} = 3^{3x-4}$

$2x = 3x-4$

$4 = x$

2. $\left(\frac{1}{4}\right)^{2x-1} = \left(\frac{1}{8}\right)^{11-x}$

$\left(\frac{1}{2}^{2}\right)^{2x-1} = \left(\frac{1}{2}^{3}\right)^{11-x}$

$\left(\frac{1}{2}\right)^{4x-2} = \left(\frac{1}{2}\right)^{33-3x}$

$4x-2 = 33-3x$

$7x = 35$

$x = 5$

3. $4^{3x-2} = \frac{1}{2}^{2x}$

$(2^{2})^{3x-2} = (2^{-1})^{2x}$

$2^{6x-4} = 2^{-2x}$

$6x-4 = -2x$

$8x = 4$

$x = \frac{1}{2}$

4. $\frac{2e^{2x}}{2} + \frac{12e^{x}}{2} - \frac{54}{2} = \frac{0}{2}$

$e^{2x} + 6e^{x} - 27 = 0$

$(e^{x}+9)(e^{x}-3) = 0$

$e^{x} = -9 \qquad e^{x} = 3$

$\cancel{\ln(-9)} \qquad \boxed{\ln(3)}$

5. $9^{2x} = 12$

$\ln_9 12 = 2x$

$\frac{\ln_9 12}{2} = x$

$0.57 = x$

6. $2.4e^{x-6} = 9.3$

$e^{x-6} = 3.875$

$\ln 3.875 = x-6$

$\ln(3.875) + 6 = x$

$7.35 = x$

7. $3^{2x} = 6^{x-1}$

$\log(3^{2x}) = \log(6^{x-1})$

$(2x)\log 3 = (x-1)\log 6$

$0.954x = 0.778x - 0.778$

$0.176x = -0.778$

$x = -4.42$

8. $e^{19x} = 23$

$\ln 23 = 19x$

$\frac{\ln(23)}{19} = x$

$0.17 = x$

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3-5 Study Guide and Intervention *(continued)*
Modeling with Nonlinear Regression

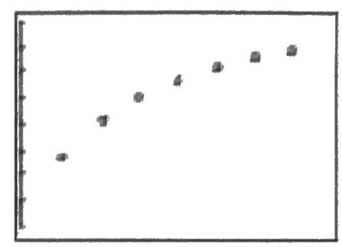
Linearizing Data The correlation coefficient is a measure calculated from a *linear* regression. Data can be **linearized** in order to measure a correlation coefficient for nonlinear data. This is accomplished by graphing $(x, r(x))$, where $r(x)$ is the regression model you are using, such as power, exponential, or logarithmic. If the data appears to be in a cluster about a line, then this regression model is likely to be a good fit. To linearize data modeled by:

- a quadratic function $y = ax^2 + bx + c$, graph $(x, \sqrt{y})$.
- an exponential function $y = ab^x$, graph $(x, \ln y)$.
- a logarithmic function $y = a \ln x + b$, graph $(\ln x, y)$.
- a power function $y = ax^b$, graph $(\ln x, \ln y)$.

Example: The spread of a given computer virus is measured over time. What model will best describe its growth? How many machines would be infected on day 9?

Day	2	4	6	8	10	12	14
Machines	765	916	1004	1066	1115	1155	1188

Step 1 Make a scatter plot of this data to get a sense of the shape.
The shape of the graph suggests a logarithmic function.



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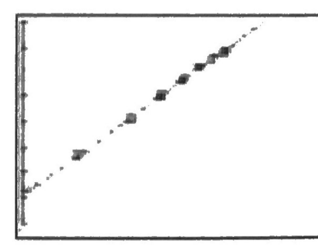
Step 2 Create a graph for $(\ln x, y)$.

ln (day)	0.693	1.386	1.792	2.079	2.303	2.485	2.639
Machines	765	916	1004	1066	1115	1155	1188

Make a scatter plot of this data and draw a line through it. This creates a straight line. Therefore, the original function is logarithmic.



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$y = 217x + 614$

$y = 217 \ln x + 614$

On day 9, there are $y = 217 \ln (9) + 614 = 1091$ infected machines.

Exercise

The brightness of a point of light is measured at different distances.

Distance (m)	20	30	40	50	60	70	80
Brightness (candelas)	18.36	8.16	4.59	2.94	2.04	1.50	1.15

a. Make a scatter plot of the data and describe the possible model.

exponential decay

~~**b.** Create a graph that linearizes the data.~~

c. Calculate an exponential or logarithmic regression model for the data and estimate the brightness at 100 meters.

$y = 33.149\,(0.956)^x$

$y = 33.149\,(0.956)^{100} = 0.035$ candelas

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3-5 Study Guide and Intervention

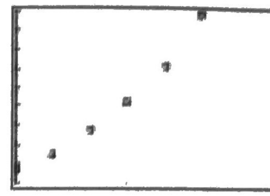
Modeling with Nonlinear Regression

Exponential, Logarithmic, and Logistic Modeling Regression can be used to model real-world data that exhibits exponential or logarithmic growth or decay.

Example: Use the data in the table to determine a regression equation that best relates the gross domestic product, GDP, (in billions of chained 2000 dollars) to the year. Estimate the 2020 GDP.

Year	1950	1960	1970	1980	1990	2000
GDP	1777.30	2501.80	3771.90	5161.70	7112.50	9817.00

Source: U.S. Dept. of Commerce



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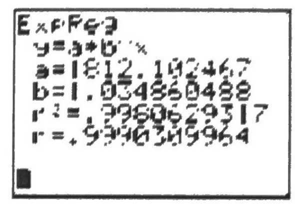
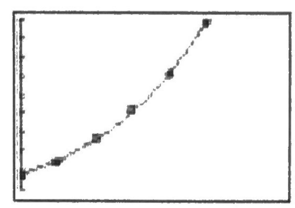
Step 1 Make a scatter plot.

Step 2 Find a function to model the data. The graph appears to be rapidly rising. Try an exponential function. With the diagnostic feature turned on and using **ExpReg** from the list of regression models yields the values shown in the second screen. The GDP in 1950 is represented by *a*, and the growth rate, 3.5% per year, is represented by $1 - b$. Notice that the correlation coefficient $r \approx 0.999$ is close to 1, indicating a close fit to the data.



Step 3 Graph the formula and the scatter plot on the same screen. In the $\boxed{Y=}$ menu, pick up this regression equation by entering $\boxed{\text{VARS}}$, **Statistics, EQ.** The exponential fit is nearly perfect.

Step 4 Estimate the 2020 GDP. Depending on the fit, it should be around 19,948.



[0, 70] scl: 10 by [1000, 10,000] scl: 1000

Exercises

The data shows the number of mice per thousand who were born with a certain genetic characteristic over the last 7 years.

x	*y*
1	4.836
2	5.721
3	6.618
4	7.489
5	8.297
6	9.019
7	9.639

1. Make a scatter plot of the data.

2. Find a logistic function to model the data.

$4.335 + 2.496 \ln x$

3. Find the value of the model at $x = 20$.

$4.335 + 2.496 \ln 20 = 11.812$

4. Make a conjecture about why a logistic model may fit the data

mice cannot keep growing. Eventually food will run out and the population will stabilize

3-4 Study Guide and Intervention *(continued)*

Exponential and Logarithmic Equations

Solve Logarithmic Equations One-to-One Property of Exponential Functions
For $b > 0$ and $b \neq 1$, $\log_b x = \log_b y$ if and only if $x = y$.

This property will help you solve logarithmic equations. For example, you can express both sides of the equation as a logarithm with the same base. Then convert both sides to exponential form, set the exponents equal to each other and solve.

Example 1: Solve $2 \log_5 4x - 1 = 11$.

$2 \log_5 4x - 1 = 11$	Original equation
$2 \log_5 4x = 12$	Add 1 to each side.
$\log_5 4x = 6$	Divide each side by 2.
$4x = 5^6$	Write in exponential form. (Use 5 as the base when exponentiating.)
$x = \frac{5^6}{4}$	Divide each side by 4.
$x = 3906.25$	Use a calculator.

Example 2: Solve $\log_2(x - 6) = 5 - \log_2 2x$.

$\log_2 (x - 6) = 5 - \log_2 2x$	Original equation
$\log_2 (x - 6) + \log_2 2x = 5$	Rearrange the logs.
$\log_2 (2x(x - 6)) = 5$	Product Property
$2x(x - 6) = 2^5$	Rewrite in exponential form.
$2x^2 - 12x - 32 = 0$	Expand.
$2(x - 8)(x + 2) = 0$	Factor.
$x = 8$ or -2	Solve.

CHECK

$x = -2$ $\log_2 (-2 - 6) = 5 - \log_2 [2(-2)]$
yields logs of negative numbers.
Therefore, -2 is extraneous.
$x = 8$ $\log_2 (8 - 6) = 5 - \log_2 [2(8)]$
$\log_2 2 = 5 - \log_2 16$, which is true.
Therefore, $x = 8$.

Exercises

Solve each logarithmic equation.

1. $\log 3x = \log 12$

$x = 4$

2. $\log_{12} 2 + \log_{12} x = \log_{12} (x + 7)$

$x = 7$

3. $\log (x + 1) + \log (x - 3) = \log (6x^2 - 6)$

$x = \frac{3}{5}$

4. $\log_3 3x = \log_3 36$

$x = 12$

5. $\log (16x + 2) + \log (20x - 2) = \log (319x^2 + 9x - 2)$

$x = 2$, $\cancel{x = -1}$

6. $\ln x + \ln (x + 16) = \ln 8 + \ln (x + 6)$

$x = 4$

1. $\log 3x = \log 12$

$3x = 12$

$x = 4$

2. $\log_{12} 2 + \log_{12} x = \log_{12}(x+7)$

$\log_{12} 2x = \log_{12}(x+7)$

$2x = x + 7$

$x = 7$

3. $\log(x+1) + \log(x-3) = \log(6x^2 - 6)$

$\log(x^2 - 2x - 3) = \log(6x^2 - 6)$

$x^2 - 2x - 3 = 6x^2 - 6$

$0 = 5x^2 + 2x - 3$

	x	1
$5x$	$5x^2$	$5x$
-3	$-3x$	-3

-15

$\wedge$

$5 + -3 = 2$

$0 = (5x-3)(x+1)$

$\boxed{x = \frac{3}{5}}$ $\cancel{x = -1}$ ← makes $\log(x+1) \rightarrow \log(-1+1) = \log(0)$

4. $\log_3 3x = \log_3 36$

$3x = 36$

$x = 12$

5. $\log(16x+2) + \log(20x-2) = \log(319x^2 + 9x - 2)$

$\log(320x^2 + 8x - 4) = \log(319x^2 + 9x - 2)$

$320x^2 + 8x - 4 = 319x^2 + 9x - 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\boxed{x = 2}$ $\cancel{x = -1}$ ← can't take log of (−)#

6. $\ln x + \ln(x+16) = \ln 8 + \ln(x+6)$

$\ln(x^2 + 16x) = \ln(8x + 48)$

$x^2 + 16x = 8x + 48$

$x^2 + 8x - 48 = 0$ $(x+12)(x-4) = 0$

$\cancel{x = -12}$ no logs of (−)numbers

$\boxed{x = 4}$