$$\cot^2\theta + 1 = \csc^2\theta$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$\cos^2 x + \sin^2 x = 1$$

Trig Identities card sort for 5.1 examples 5-7

| | |
|---|---|
| **#1    Simplify**  $$\dfrac{1}{\cos x}(1 - \sin^2 x)$$ | $\dfrac{1}{\cos x} \cdot \cos^2 x \quad \boxed{= \cos x}$ |
| **#2    Simplify**  $$\csc x \sec x - \tan x$$ | $\dfrac{1}{\sin x} \cdot \dfrac{1}{\cos x} - \dfrac{\sin x}{\cos x} \cdot \dfrac{\sin x}{\sin x}$   $\dfrac{1 - \sin^2 x}{\sin x \cos x} = \dfrac{\cos^2 x}{\sin x \cos x} = \dfrac{\cos x}{\sin x}$   $\boxed{= \cot x}$ |
| **#3    Simplify**  $$\dfrac{1 - \sin^2 x}{\csc^2 x - 1}$$ | $\dfrac{\cos^2 x}{\cot^2 x} = \dfrac{\cos^2 x}{\frac{\cos^2 x}{\sin^2 x}}$   $= \cos^2 x \cdot \dfrac{\sin^2 x}{\cos^2 x}$   $\boxed{= \sin^2 x}$ |
| **#4    Simplify**  $$\dfrac{\sec x \csc x - \tan x}{\sec x \csc x}$$ | $1 - \dfrac{\tan x}{\sec x \csc x}$   $1 - \dfrac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} \cdot \frac{1}{\sin x}} = 1 - \dfrac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x \sin x}}$   $= 1 - \dfrac{\sin x}{\cos x} \cdot \dfrac{\cos x \sin x}{1}$   $= 1 - \sin^2 x = \boxed{\cos^2 x}$ |
| **#5    Simplify**  $\dfrac{\csc x - 1}{\csc x - 1} \cdot \dfrac{\csc x - 1}{\sin x}$ $$\dfrac{\sin x}{\csc x + 1} \cdot \dfrac{\csc x + 1}{} + \dfrac{\sin x}{\csc x - 1} \cdot \dfrac{\csc x + 1}{\csc x + 1}$$ | $\dfrac{\csc x \sin x - \sin x + \sin x \csc x + \sin x}{\csc^2 x - 1}$   $\dfrac{2}{\cot^2\theta} \cdot \boxed{2\tan^2\theta}$ |

$\sec x \cdot \cos x$
$1 \cdot \cos x$
$\cos x$

**#6 Simplify**

$\sec x - 1$
$\sec x - 1$

$$\frac{\cos x}{\sec x + 1} + \frac{\cos x}{\sec x - 1} \cdot \frac{\sec x + 1}{\sec x + 1}$$

$$\frac{\sec x \cos x - \cos x + \cos x \sec x + \cos x}{\sec^2 x - 1}$$

$$\frac{2}{\tan^2 \theta} = \boxed{2\cot^2 \theta}$$

---

**#7 Simplify**

$$\cos x \tan x - \sin x \cos^2 x$$

$$\cos x \cdot \frac{\sin x}{\cos x} - \sin x \cos^2 x$$

$$\sin x - \sin x \cos^2 x$$

$$\sin x (1 - \cos^2 x)$$

$$\sin x (\sin^2 x) = \boxed{\sin^3 x}$$

---

**#8 Rewrite as an expression not containing a fraction.**

$$\frac{\sin x}{\csc x - \cot x} \cdot \frac{\csc x + \cot x}{\csc x + \cot x}$$

$\sin x \cdot \cot x$
$\sin x \cdot \frac{\cos x}{\sin x}$

$$\frac{\sin x \csc x + \sin x \cot x}{\csc^2 x - \cot^2 x}$$

$$\frac{1 + \cos x}{\cot^2 x + 1 - \cot^2 x} = \frac{1 + \cos x}{1}$$

$$\boxed{1 + \cos x}$$

---

**#9 Rewrite as an expression not containing a fraction.**

$$\frac{2\sin x}{\cot x + \csc x} \cdot \frac{\cot x - \csc x}{\cot x - \csc x}$$

$$\frac{2\sin x \cot x - 2\sin x \csc x}{\cot^2 x - \csc^2 x}$$

$$\frac{2\cos x - 2}{\csc^2 x - 1 - \csc^2 x} = \frac{2\cos x - 2}{-1}$$

$$\boxed{2 - 2\cos x}$$

---

**#10 Simplify**

$$-\csc\left(\frac{\pi}{2} - x\right) - \tan^2 x \sec x$$

$$-\sec x - \tan^2 x \sec x$$

$$-\sec x (1 + \tan^2 x)$$

$$-\sec x (\sec^2 x)$$

$$\boxed{-\sec^3 x}$$