

Released Items

Student Name: Key

Fall 2015
NC Final Exam
Precalculus



Student Booklet



Public Schools of North Carolina
State Board of Education
Department of Public Instruction
Raleigh, North Carolina 27699-6314



1 What transformations have occurred to create the function $f(x) = 3x^3 - 4$ from the function $g(x) = x^3$? *vertical stretch*
down 4

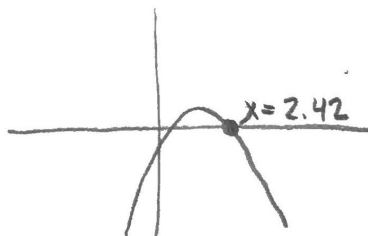
- A The graph of the function has been stretched horizontally and shifted up four units.
- B The graph of the function has been stretched vertically and shifted up four units.
- C The graph of the function has been stretched horizontally and shifted down four units.
- D The graph of the function has been stretched vertically and shifted down four units.

2 An object is launched straight upward from ground level with an initial velocity of 50.0 feet per second. The height, h (in feet above ground level), of the object t seconds after the launch is given by the function $h(t) = -16t^2 + 50t$. At **approximately** what value of t will the object have a height of 28.0 feet and be traveling downward?

- A 2.39 seconds
- B 1.84 seconds
- C 1.56 seconds
- D 0.73 seconds

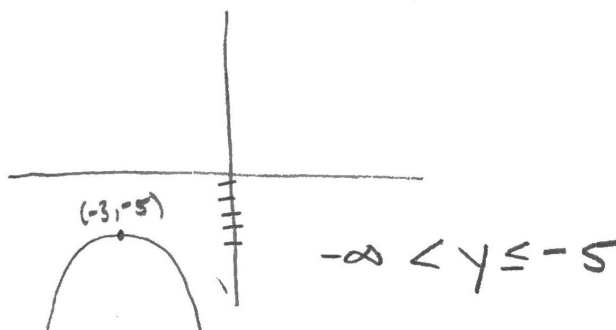
$$28 = -16t^2 + 50t$$

$$0 = -16t^2 + 50t - 28$$



3 What is the range of the function $f(x) = -5 - 2(x + 3)^2$?

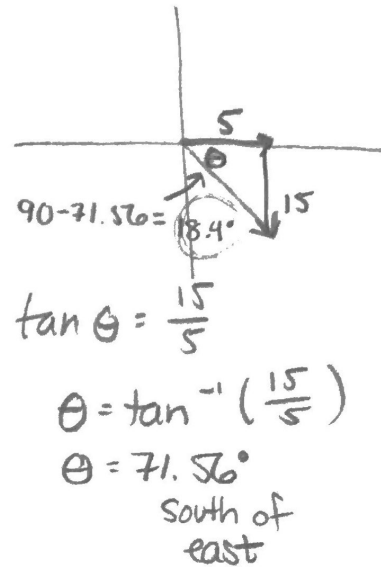
- A $[-5, \infty)$
- B $(-\infty, 5]$
- C $(-\infty, -5]$
- D $(-\infty, \infty)$





4 A wind that is blowing from the northwest toward the southeast can be represented by a vector. The vector has an eastward component and a southward component. If the eastward component has a magnitude of 5.00 miles per hour and the southward component has a magnitude of 15.00 miles per hour, in what direction is the wind blowing?

- A The wind is blowing in the direction 71.6° east of south.
- B The wind is blowing in the direction 67.5° east of south.
- C The wind is blowing in the direction 22.5° east of south.
- D The wind is blowing in the direction 18.4° east of south.



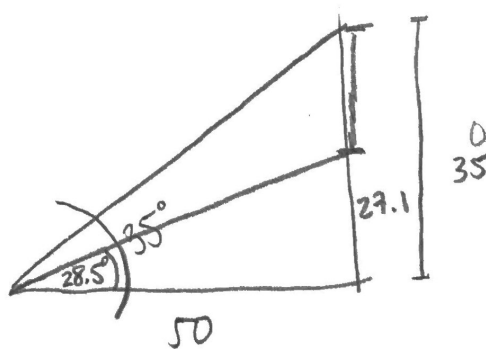
5 What value of x satisfies the equation $\log_3(x - 4) = 2$?

- A 5
- B 10
- C 12
- D 13

$3^2 = x - 4$
 $9 = x - 4$
 $13 = x$

6 A man is standing on level ground 50 feet away from the wall of a building. He looks up at a window on the building. The angle of elevation to the bottom of the window is 28.5° . He then looks up at the top of the building. The angle of elevation to the top of the building is 35° . What is the **approximate** distance between the bottom of the window and the top of the building?

- A 5.7 feet
- B 7.9 feet
- C 8.3 feet
- D 8.5 feet



$\tan(35^\circ) = \frac{x}{50}$
 $x = 35$
 $\tan(28.5^\circ) = \frac{y}{50}$ $y = 27.1$
 $35 - 27.1 = 7.9$

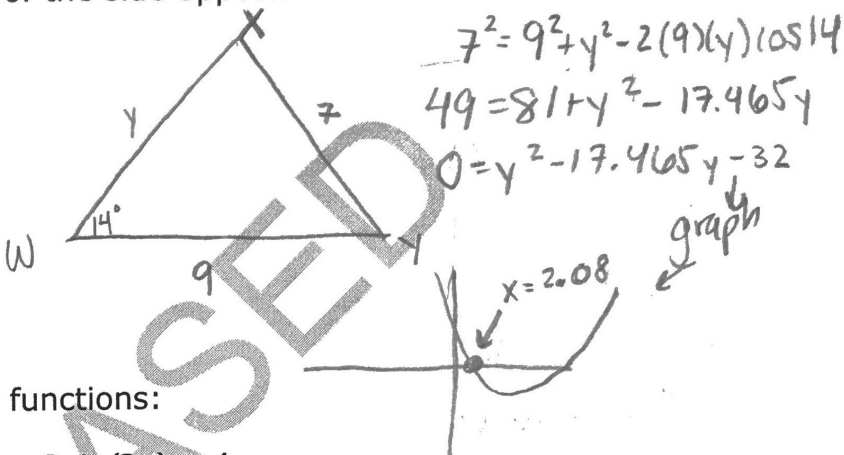


7 Triangle WXY has the following properties:

- The angle at vertex W is 14° , and the angle at vertex X is obtuse.
- The side opposite vertex W has a length of 7.00 units.
- The side opposite vertex X has a length of 9.00 units.

What is the **approximate** length of the side opposite vertex Y ?

- A 1.73 units
- B 2.08 units
- C 3.26 units
- D 5.40 units



8 Consider these two trigonometric functions:

$$f(x) = 3 \sin(2x) + 4$$

$$g(x) = 3 \sin\left(2x - \frac{\pi}{2}\right) + 4$$

How should the graph of f be shifted to produce the graph of g ?

- A Shift the graph of f to the left $\frac{\pi}{4}$ units to produce the graph of g .
- B Shift the graph of f to the right $\frac{\pi}{4}$ units to produce the graph of g .
- C Shift the graph of f to the left $\frac{\pi}{2}$ units to produce the graph of g .
- D Shift the graph of f to the right $\frac{\pi}{2}$ units to produce the graph of g .

phase shift = $\frac{c}{b}$
 $= \frac{\frac{\pi}{2}}{2}$
 $= \frac{\pi}{4}$



- 9 The maximum height, in inches, a ball reaches after its first four bounces is shown in the table below.

Bounce Number	Height (in inches)
1	42.0
2	31.5
3	23.6
4	17.7

$$\frac{31.5}{42} = 0.75$$

$$\frac{23.6}{31.5} = 0.749$$

↑
keeping 75%
of height

Which type of function **best** models the data and why?

- A an exponential function, because the height of the ball is decreasing by 25% with each bounce
- B an exponential function, because the height of the ball is decreasing by 75% with each bounce
- C a logistic function, because the height of the ball is decreasing by 25% with each bounce
- D a logistic function, because the height of the ball is decreasing by 75% with each bounce
- 10 What is the inverse function of $g(x) = x^3 - 2$?

A $g^{-1}(x) = \sqrt[3]{x + 2}$

$$y = x^3 - 2$$

B $g^{-1}(x) = \sqrt[3]{x - 2}$

$$x = y^3 - 2$$

C $g^{-1}(x) = \sqrt[3]{x} + 2$

$$x + 2 = y^3$$

D $g^{-1}(x) = \left(\frac{x - 2}{3}\right)^3$

$$\sqrt[3]{x + 2} = y$$



11 What are the polar coordinates of the point $(-2\sqrt{3}, 2\sqrt{3})$, where $0 \leq \theta \leq 360^\circ$?

A $(2\sqrt{6}, 150^\circ)$ and $(-2\sqrt{6}, 210^\circ)$

B $(2\sqrt{6}, 135^\circ)$ and $(-2\sqrt{6}, 315^\circ)$

C $(2\sqrt{6}, 120^\circ)$ and $(-2\sqrt{6}, 240^\circ)$

D $(2\sqrt{6}, 30^\circ)$ and $(-2\sqrt{6}, 330^\circ)$

$$r^2 = x^2 + y^2$$

$$r^2 = (-2\sqrt{3})^2 + (2\sqrt{3})^2$$

$$r^2 = 4 \cdot 3 + 4 \cdot 3$$

$$r^2 = 24$$

$$r = \sqrt{24} = 2\sqrt{6}$$

$$\theta = \tan^{-1}\left(\frac{2\sqrt{3}}{-2\sqrt{3}}\right)$$

$$\theta = \tan^{-1}(-1)$$

$$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$\downarrow \qquad \downarrow$$

$$135^\circ \qquad 315^\circ$$

12 Which equation is the rectangular form of the polar equation $r = \frac{2}{1 + \cos \theta}$?

A $x^2 + 4y = 4$

B $x^2 + y^2 = 4$

C $y^2 + 4x = 4$

D $y^2 - 4x = 4$

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$$(1 + \cos \theta) \cdot r = \frac{2}{1 + \cos \theta} (1 + \cos \theta)$$

$$r + r \cos \theta = 2$$

$$\sqrt{x^2 + y^2} + x = 2$$

$$\sqrt{x^2 + y^2} = 2 - x$$

$$x^2 + y^2 = 4 - 4x + x^2$$

$$y^2 = 4 - 4x$$

$$y^2 + 4x = 4$$

PRECALCULUS — RELEASED ITEMS



- 13 Two parametric equations are shown below, where $t \geq 0$.

$$\begin{aligned} x &= \frac{1}{3}\sqrt{t} + 3 & x-3 &= \frac{1}{3}\sqrt{t} \\ y &= 4t^2 - 7 & 3x-9 &= \sqrt{t} \\ & & (3x-9)^2 &= t \end{aligned}$$

Which nonparametric equation can be used to graph the curve described by the parametric equations?

A $y = \frac{4}{9}(x + 1) - 7$

B $y = \frac{4}{3}(x + 3) - 7$

C $y = 36(x - 1)^4 - 7$

D $y = 324(x - 3)^4 - 7$

$$y = 4((3x-9)^2)^2 - 7$$

$$y = 4((3(x-3))^2)^2 - 7$$

$$y = 4(9(x-3)^2)^2 - 7$$

$$y = 4(81(x-3)^4) - 7$$

$$y = 324(x-3)^4 - 7$$

- 14 The formula for a sequence is shown below.

$$a_n = 2a_{n-1} + 3, a_1 = 3$$

3, 9, 21, ...

$$2(9) + 3 = 21$$

$$2(3) + 3 = 9$$

Which is another formula that represents the sequence?

A $f(n) = 3(2^n - 1)$

$$3(2^1 - 1) = 3(1) = 3$$

$$3(2^2 - 1) = 3(3) = 9$$

~~B~~ $f(n) = 2n^3 - 3n^2 + 8n + 3$ $2(1)^3 - 3(1)^2 + 8(1) + 3 = 10$

~~C~~ $f(n) = 2(n^2 + 1)$ $2(1^2 + 1) = 2(2) = 4$

~~D~~ $f(n) = 3n^2 + 8n - 1$ $3(1)^2 + 8(1) - 1 = 10$



15 When $a_1 = 25,000$, what is the sum of the infinite sequence defined by the equation $a_{n+1} = 0.8a_n$?

- A 125,000
- B 140,000
- C 160,000
- D 195,000

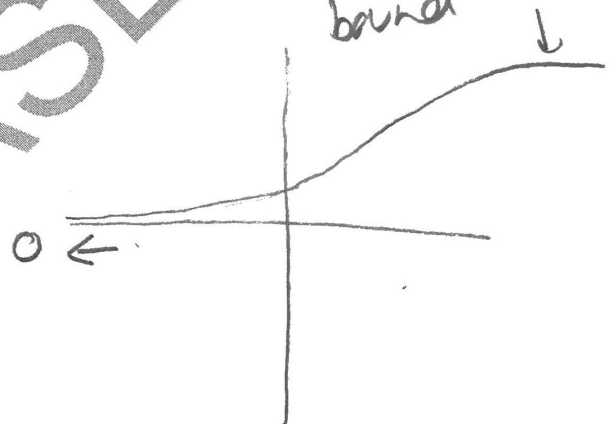
$$S = \frac{a}{1-r}$$

$$S = \frac{25,000}{1-0.8}$$

$$S = 125,000$$

16 What is the end behavior of the function $f(x) = \frac{100}{1 + 5(0.75)^x}$? ← logistic, so it has an upper bound

- A $\lim_{x \rightarrow -\infty} f(x) = 0$ and $\lim_{x \rightarrow \infty} f(x) = \infty$
- B $\lim_{x \rightarrow -\infty} f(x) = 0$ and $\lim_{x \rightarrow \infty} f(x) = 100$
- C $\lim_{x \rightarrow -\infty} f(x) = 1$ and $\lim_{x \rightarrow \infty} f(x) = \infty$
- D $\lim_{x \rightarrow -\infty} f(x) = 1$ and $\lim_{x \rightarrow \infty} f(x) = 100$



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- 17 In the piecewise function below, k is a constant.

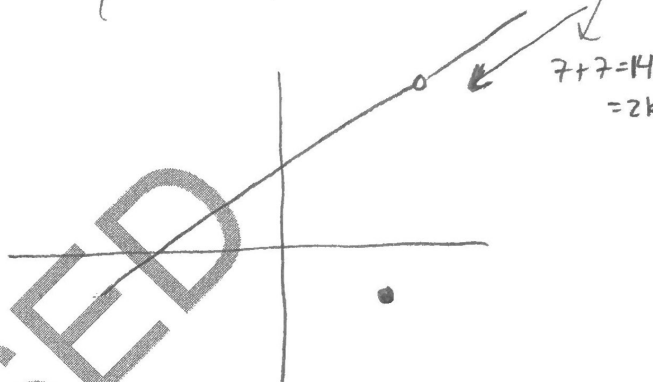
$$f(x) = \begin{cases} \frac{x^2 - k^2}{x - k}, & x \neq k \\ 4 - k, & x = k \end{cases}$$

PICK a # for k ; $k = 7$

$\frac{x^2 - 49}{x - 7}, x \neq 7 \rightarrow \frac{(x-7)(x+7)}{x-7}$
 $4 - 7, x = 7$

What is the value of the limit $\lim_{x \rightarrow k^-} f(x)$?

- A $-2k$
- B $2k$
- C 0
- D Limit does not exist.



- 18 What is the value of $\lim_{x \rightarrow 3} (x^2 - 3x + 7)$?

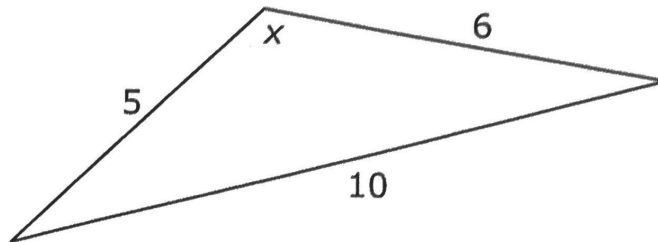
- A -2
- B 7
- C 25
- D Limit does not exist.

continuous everywhere so

$$\lim_{x \rightarrow 3} (3^2 - 3(3) + 7) = 7$$



- 19 What is the **approximate** measure of angle x in the triangle below?



- A 60.3°
- B 80.4°
- C 117.1°
- D 130.5°

$$10^2 = 5^2 + 6^2 - 2(5)(6)\cos x$$

$$100 = 61 - 60\cos x$$

$$39 = -60\cos x$$

$$-0.65 = \cos x$$

$$\cos^{-1}(-0.65) = x \quad x = 130.5^\circ$$

- 20 The temperature, in degrees F, of the water in a large fish tank is modeled by the function $T(x) = \ln(1 + x) + 52.4$, where x is the number of pebbles in the tank. **Approximately** how many pebbles are in the tank if the water is 58.3°F?

- A 360
- B 300
- C 270
- D 200

$$58.3 = \ln(1+x) + 52.4$$

$$5.9 = \ln_e(1+x)$$

$$e^{5.9} = 1+x$$

$$e^{5.9} - 1 = x$$

$$364.03 = x$$



21 A series is shown below.

$$\frac{1}{1} + \frac{2}{5} + \frac{4}{25} + \frac{8}{125} + \dots + \frac{16}{625}$$

Which statement is true about the sum of the series?

~~A~~ The series converges to $\frac{7}{3} = 2.33$

$$1 + \frac{2}{5} + \frac{4}{25} + \frac{8}{125} + \frac{16}{625} = 1.6496$$

~~B~~ The series converges to $\frac{5}{2} = 2.5$

C The series converges to $\frac{5}{3} = 1.666$

D The series diverges.

22 A circle is graphed using the parametric equations shown below.

$$x = 5\cos(t) + 3$$

$$\frac{x-3}{5} = \cos(t)$$

$$y = 5\sin(t) - 1$$

$$\frac{y+1}{5} = \sin(t)$$

Where is the center of the circle located?

A (-3, -1)

B (-3, 1)

C (3, -1)

D (3, 1)

$$\cos^2 t + \sin^2 t = 1$$

$$\frac{(x-3)^2}{25} + \frac{(y+1)^2}{25} = 1$$

$$(3, -1)$$



- 23 The polar coordinates of a point are $(6, \frac{4\pi}{3})$. What are the rectangular coordinates of the point?

A $(3, -3\sqrt{3})$

B $(3, 3\sqrt{3})$

C $(-3, -3\sqrt{3})$

D $(-3, 3\sqrt{3})$

$$x = r \cos \theta$$

$$x = 6 \cos \left(\frac{4\pi}{3} \right)$$

$$x = 6 \left(-\frac{1}{2} \right)$$

$$x = -3$$

$$y = r \sin \theta$$

$$y = 6 \sin \frac{4\pi}{3}$$

$$y = 6 \left(-\frac{\sqrt{3}}{2} \right)$$

$$y = -3\sqrt{3}$$

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