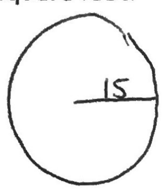
FUNCTION COMPOSITIONS

Since functions convert the value of an input variable into the value of an output variable, it stands to reason that this output could then be used as an input to a second function. This process is known as composition of functions, in other words, combining the action or rules of two functions.

***Exercise* #1:** A circular garden with a radius of 15 feet is to be covered with topsoil at a cost of $1.25 per square foot of garden space.

$A = \pi r^2$

(a) Determine the area of this garden to the nearest square foot.



$A = \pi(15)^2$

$= 707 \text{ ft}^2$

(b) Using your answer from (a), calculate the cost of covering the garden with topsoil.

$\$1.25 \times 707 \text{ ft}^2 = \$883.75$

In this exercise, we see that the output of an area function is used as the input to a cost function. This idea can be generalized to generic functions, *f* and *g* as shown in the diagram below.

Output from *f* becomes the input to *g*

Input = x → f → g → ***Final output = y***

There are two notations that are used to indicate composition of two functions. These will be introduced in the next few exercises, both with equations and graphs.

***Exercise* #2:** Given $f(x) = x^2 - 5$ and $g(x) = 2x + 3$, find values for each of the following.

(a) $f(g(1)) = 20$

$g(1) = 2(1) + 3 = 5$

$f(5) = (5)^2 - 5 = 20$

(b) $g(f(2)) = 1$

$f(2) = (2)^2 - 5 = -1$

$g(-1) = 2(-1) + 3 = 1$

(c) $g(g(0)) = 9$

$g(0) = 2(0) + 3 = 3$

$g(3) = 2(3) + 3 = 9$

(d) $(f \circ g)(-2) = -4$

$f(g(-2))$

$g(-2) = 2(-2) + 3 = -1$

$f(-1) = (-1)^2 - 5 = -4$

(e) $(g \circ f)(3) = 11$

$f(3) = (3)^2 - 5 = 4$

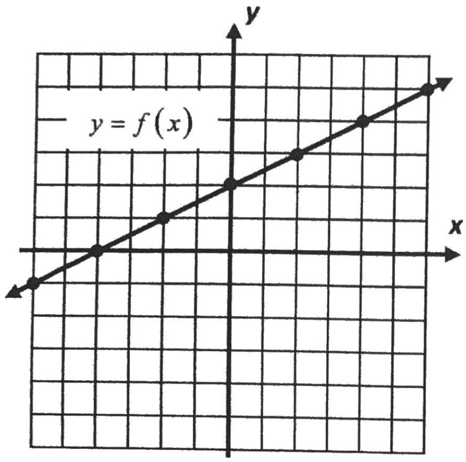
$g(4) = 2(4) + 3 = 11$

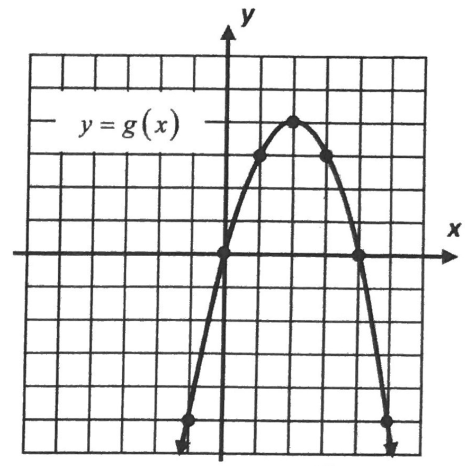
(f) $(f \circ f)(-1) = 11$

$f(-1) = (-1)^2 - 5 = -4$

$f(-4) = (-4)^2 - 5 = 11$

Exercise #3: The graphs below are of the functions $y = f(x)$ and $y = g(x)$. Evaluate each of the following questions based on these two graphs.





(a) $g(f(2)) = 3$ ($f(2) = 3$)

(b) $f(g(-1)) = -0.5$ ($g(-1) = -5$)

(c) $g(g(1)) = 3$ ($g(1) = 3$)

(d) $(g \circ f)(-2) = 3$ ($f(-2) = 1$)

(e) $(f \circ g)(0) = 2$ ($g(0) = 0$)

(f) $(f \circ f)(0) = 3$ ($f(0) = 2$)

On occasion, it is desirable to create a formula for the composition of two functions. We will see this facet of composition throughout the course as we study functions. The next two exercises illustrate the process of finding these equations with simple linear and quadratic functions.

Exercise #4: Given the functions $f(x) = 3x - 2$ and $g(x) = 5x + 4$, determine formulas in simplest $y = ax + b$ form for:

(a) $f(g(x))$

$f(5x+4) = 3(5x+4) - 2$

$15x + 12 - 2$

$15x + 10$

(b) $g(f(x))$

$g(3x-2) = 5(3x-2) + 4$

$15x - 10 + 4$

$15x - 6$

Exercise #5: If $f(x) = x^2$ and $g(x) = x - 5$ then $f(g(x)) =$

$f(x-5) = (x-5)^2$

$= x^2 - 10x + 25$

	x	-5
x	x^2	$-5x$
-5	$-5x$	25