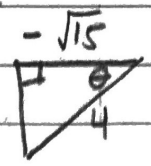
pg. 359 #1-30 all

11/26/18

1. $\csc\theta = -4$

$\sin\theta = -\frac{1}{4}$

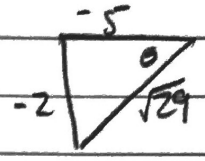
$\cos\theta = -\frac{\sqrt{15}}{4}$




2. $\tan\theta = \frac{2}{5}$

$\csc\theta = -\frac{\sqrt{29}}{2}$

$\sec\theta = -\frac{\sqrt{29}}{5}$




3. $\frac{\sin(90°-x)}{\tan(90°-x)} = \frac{\cos(x)}{\cot(x)} = \cos(x) \cdot \frac{\sin(x)}{\cos(x)} = \sin(x)$

4. $\frac{\sec^2 x - 1}{\tan^2 x + 1} = \frac{\tan^2 x}{\sec^2 x} = \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}} = \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{1} = \sin^2 x$

5. $\sin\theta(1+\cot^2\theta)$

$\sin\theta(\csc^2\theta)$

$\sin\theta \cdot \frac{1}{\sin^2\theta} = \frac{1}{\sin\theta} = \csc\theta$

6. $\frac{\csc^2\theta - 1}{\csc^2\theta} + \frac{\sec^2\theta - 1}{\sec^2\theta} = 1$

$\frac{\cot^2\theta}{\csc^2\theta} + \frac{\tan^2\theta}{\sec^2\theta} = 1$

$\frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{1}{\sin^2\theta}} + \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} = 1$

$\cos^2\theta + \sin^2\theta = 1$

$1 = 1$ ✓

$\frac{\cos\theta}{\cos\theta}\cdot\frac{(1+\sin\theta)}{1+\sin\theta} + \frac{1-\sin\theta}{\cos\theta}\cdot\frac{(1+\sin\theta)}{(1+\sin\theta)} = \frac{2\cos\theta}{1+\sin\theta}$

$\frac{\cos^2\theta}{\cos\theta(1+\sin\theta)} + \frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)} = \frac{2\cos\theta}{1+\sin\theta}$

$\frac{\cos^2\theta + \cos^2\theta}{\cos\theta(1+\sin\theta)} = \frac{2\cos\theta}{1+\sin\theta}$

$\frac{2\cos^2\theta}{\cos\theta(1+\sin\theta)} = \frac{2\cos\theta}{1+\sin\theta}$

$\frac{2\cos\theta}{1+\sin\theta} = \frac{2\cos\theta}{1+\sin\theta}$ ✓

8. $\frac{1}{1+\cos\theta}\cdot\frac{(1-\cos\theta)}{(1-\cos\theta)} + \frac{1}{1-\cos\theta}\cdot\frac{(1+\cos\theta)}{(1+\cos\theta)} = 2\csc^2\theta$

$\frac{1-\cos\theta + 1+\cos\theta}{1-\cos^2\theta} = 2\csc^2\theta$

$\frac{2}{\sin^2\theta} = 2\csc^2\theta$

$2\csc^2\theta = 2\csc^2\theta$ ✓

9. $-\sec^2\theta \sin^2\theta = \frac{\cos^2\theta - 1}{\cos^2\theta}$

$-\frac{1}{\cos^2\theta} \cdot \sin^2\theta = \frac{\cos^2\theta - 1}{\cos^2\theta}$

$\frac{-\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta - 1}{\cos^2\theta}$

$\frac{-(1-\cos^2\theta)}{\cos^2\theta} = \frac{\cos^2\theta - 1}{\cos^2\theta}$

$\frac{-1+\cos^2\theta}{\cos^2\theta} = \frac{\cos^2\theta - 1}{\cos^2\theta}$ ✓

10. $\sin^4 x - \cos^4 x = 2\sin^2 x - 1$

$(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = 2\sin^2 x - 1$

$\sin^2 x - \cos^2 x = 2\sin^2 x - 1$

$\sin^2 x - (1 - \sin^2 x) = 2\sin^2 x - 1$

$2\sin^2 x - 1 = 2\sin^2 x - 1$ ✓

11. D: only true for $\tan^2\theta$ and $\sec^2\theta$

12. $\sqrt{2}\sin\theta + 1 = 0$

$\sin\theta = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ $\quad \theta = \frac{5\pi}{4}, \frac{7\pi}{4}$

13. $\sec^2\theta = \frac{4}{3}$

$\sec\theta = \pm\frac{2}{\sqrt{3}}$

$\cos\theta = \pm\frac{\sqrt{3}}{2}$ $\quad \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

14. $\tan^2\theta - \tan\theta = 0$

$\tan\theta(\tan\theta - 1) = 0$

$\tan\theta = 0$ $\quad \tan\theta = 1$

$\theta = 0, 2\pi$ $\quad \theta = \frac{\pi}{4}, \frac{5\pi}{4}$

$\theta = 0 + 2\pi n$ $\quad \theta = \frac{\pi}{4} + \pi n$

15. $\frac{1-\sin\theta}{\cos\theta} = \cos\theta$

$1 - \sin\theta = \cos^2\theta$

$1 - \sin\theta = 1 - \sin^2\theta$

$\sin^2\theta - \sin\theta = 0$

$\sin\theta(\sin\theta - 1) = 0$

$\sin\theta = 0$ $\quad \sin\theta = 1$

$\theta = 0 + 2\pi n$ $\quad \theta = \frac{\pi}{2} + 2\pi n$

extraneous; makes $\cos\theta = 0$

16. $\frac{1}{\sec\theta - 1} - \frac{1}{\sec\theta + 1} = 2$

(multiply by $\frac{\sec\theta+1}{\sec\theta+1}$ and $\frac{\sec\theta-1}{\sec\theta-1}$)

$\frac{\sec\theta + 1 - (\sec\theta - 1)}{\sec^2\theta - 1} = 2$

$\frac{2}{\sec^2\theta - 1} = 2$

$\sec^2\theta - 1 = 1$

$\sec^2\theta = 2$

$\cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm\frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\theta = \frac{\pi}{4} + \frac{\pi}{2}n$

17. $\sec\theta - 2\tan\theta = 0$

$\frac{1}{\cos\theta} - \frac{2\sin\theta}{\cos\theta} = 0$

$\frac{1-2\sin\theta}{\cos\theta} = 0$

$1 - 2\sin\theta = 0$

$1 = 2\sin\theta$

$\frac{1}{2} = \sin\theta$

$\theta = \frac{\pi}{6} + 2\pi n, \frac{5\pi}{6} + 2\pi n$

18. $\frac{20}{40} = \frac{40\sin(135\pi t)}{40}$

$\frac{1}{2} = \sin(135\pi t)$

$\sin^{-1}\left(\frac{1}{2}\right) = 135\pi t$

$\frac{\frac{\pi}{6}}{135\pi} = \frac{135\pi t}{135\pi}$

$t = 0.0012 \text{ sec}$

19. $\tan(165) = \tan(135+30)$

$= \frac{\tan 135 + \tan 30}{1 - \tan 135 \cdot \tan 30}$

$= \frac{-1 + \frac{1/2}{\sqrt{3}/2}}{1 - (-1)\left(\frac{1/2}{\sqrt{3}/2}\right)} = \frac{-1 + \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}} = \frac{\frac{-3+\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}} = \frac{-3+\sqrt{3}}{3+\sqrt{3}} \cdot \frac{3-\sqrt{3}}{3-\sqrt{3}} = \frac{-9+6\sqrt{3}-3}{9-3}$

$= \frac{-12+6\sqrt{3}}{6}$

$= -2+\sqrt{3}$

20. $\cos\left(-\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$ $\quad \frac{\pi}{3} - \frac{\pi}{4} = \frac{3\pi}{12} - \frac{4\pi}{12} = -\frac{\pi}{12}$

$= \cos\pi/3\cos\pi/4 + \sin\pi/3\cos\pi/4$

$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$= \frac{\sqrt{2}+\sqrt{6}}{4}$

21. $\sin(75^\circ) = \sin(30+45)$

$= \sin 30\cos 45 + \cos 30 \sin 45$

$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}$

22. $\cos 465 - \cos 15$

$\cos(330+135) - \cos(45-30)$

$(\cos 330\cos 135 - \sin 330\sin 135) - (\cos 45\cos 30 + \sin 45\sin 30)$

$\left(\frac{\sqrt{3}}{2}\cdot\frac{-\sqrt{2}}{2} - \frac{-1}{2}\cdot\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}\right)$

$\left(\frac{-\sqrt{6}}{4} + \frac{\sqrt{2}}{4}\right) - \left(\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}\right)$

$\frac{-2\sqrt{6}}{4} = \frac{-\sqrt{6}}{2}$

23. $6\sin 675 - 6\sin 45$

$6\sin(360+315) - 6\sin 45$

$6(\sin 360\cos 315 + \cos 360\sin 315) - 6\sin 45$

$6\left(0\cdot\frac{\sqrt{2}}{2} + 1\cdot\frac{-\sqrt{2}}{2}\right) - 6\cdot\frac{\sqrt{2}}{2}$

$6\left(\frac{-\sqrt{2}}{2}\right) - 6\cdot\frac{\sqrt{2}}{2}$

$\frac{-6\sqrt{2}}{2} - \frac{6\sqrt{2}}{2} = \frac{-12\sqrt{2}}{2} = -6\sqrt{2}$

24. F: $\cos(\theta+\pi) \stackrel{?}{=} -\sin\pi$

$\cos(0+\pi) \stackrel{?}{=} -\sin\pi$

$\cos\pi \stackrel{?}{=} -\sin\pi$

$1 \neq 0$

G: $\cos(\pi-\theta) \stackrel{?}{=} \cos\theta$

$\cos(\pi-0) \stackrel{?}{=} \cos 0$

$\cos\pi = \cos 0$

$-1 \neq 1$

H: $\sin(\theta - \frac{3\pi}{2}) \stackrel{?}{=} \cos\theta$

$\sin(0 - \frac{3\pi}{2}) \stackrel{?}{=} \cos(0)$

$\sin(-\frac{3\pi}{2}) \stackrel{?}{=} \cos(0)$

$1 = 1$ ✓

25. $\cos\frac{\pi}{8}\cos\frac{3\pi}{8} - \sin\frac{\pi}{8}\sin\frac{3\pi}{8}$

$\cos\left(\frac{\pi}{8}+\frac{3\pi}{8}\right)$

$\cos\left(\frac{4\pi}{8}\right) = \cos\left(\frac{\pi}{2}\right) = 0$

26. $\frac{\tan 135 - \tan 15}{1+\tan 135\tan 15}$

$\tan(135-15)$

$\tan(120) = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \frac{\sqrt{3}}{2}\cdot\frac{-2}{1} = -\sqrt{3}$

27a) $\frac{v^2\sin 2\theta}{g} = \frac{2}{g}v^2(\tan\theta - \tan\theta\sin^2\theta)$

$\frac{2}{g}v^2\left(\frac{\sin\theta}{\cos\theta} - \frac{\sin\theta\cdot\sin^2\theta}{\cos\theta}\right)$

$\frac{2}{g}v^2\left(\frac{\sin\theta - \sin^3\theta}{\cos\theta}\right)$

$\frac{2}{g}v^2\left(\frac{\sin\theta(1-\sin^2\theta)}{\cos\theta}\right)$

$\frac{2}{g}v^2\left(\frac{\sin\theta\cdot\cos^2\theta}{\cos\theta}\right)$

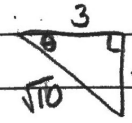
$\frac{2}{g}v^2(\sin\theta\cos\theta)$

$\frac{v^2}{g}(2\sin\theta\cos\theta)$

$\frac{v^2\sin 2\theta}{g}$ ✓

b) $\frac{\frac{v^2\sin^2\theta}{2g}}{\frac{v^2\sin 2\theta}{g}} = \frac{v^2\sin^2\theta}{2g}\cdot\frac{g}{v^2\sin 2\theta} = \frac{\sin^2\theta}{2\sin 2\theta} = \frac{\sin^2\theta}{2\cdot 2\sin\theta\cos\theta} = \frac{\sin\theta}{4\cos\theta} = \frac{1}{4}\tan\theta$

28. $\tan\theta = -3$




$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\cdot -\frac{1}{\sqrt{10}}\cdot\frac{3}{\sqrt{10}}$

$= -\frac{6}{10} = -\frac{3}{5}$

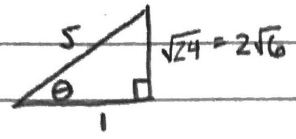
$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{3}{\sqrt{10}}\right)^2 - \left(-\frac{1}{\sqrt{10}}\right)^2$

$= \frac{9}{10} - \frac{1}{10} = \frac{8}{10} = \frac{4}{5}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2(-3)}{1-(-3)^2} = \frac{-6}{1-9} = \frac{-6}{-8} = \frac{3}{4}$

29. $\cos\theta = \frac{1}{5}$

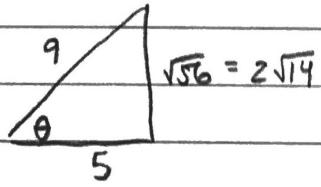



$\sin 2\theta = 2\sin\theta\cos\theta$
$= 2 \cdot \frac{2\sqrt{6}}{5} \cdot \frac{1}{5}$
$= \frac{4\sqrt{6}}{25}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \left(\frac{1}{5}\right)^2 - \left(\frac{\sqrt{24}}{5}\right)^2$
$= \frac{1}{25} - \frac{24}{25} = \frac{-23}{25}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2 \cdot \frac{2\sqrt{6}}{1}}{1-\left(\frac{2\sqrt{6}}{1}\right)^2} = \frac{4\sqrt{6}}{1-(4\cdot 6)} = \frac{4\sqrt{6}}{-23}$

30. $\cos\theta = \frac{5}{9}$




$\sin 2\theta = 2\sin\theta\cos\theta$
$= 2 \cdot \frac{2\sqrt{14}}{9} \cdot \frac{5}{9}$
$= \frac{20\sqrt{14}}{81}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \left(\frac{5}{9}\right)^2 - \left(\frac{2\sqrt{14}}{9}\right)^2$
$= \frac{25}{81} - \frac{4\cdot 14}{81} = \frac{-31}{81}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{2\sqrt{14}}{5}\right)}{1-\left(\frac{2\sqrt{14}}{5}\right)^2} = \frac{\frac{4\sqrt{14}}{5}}{1-\frac{56}{25}}$

$= \frac{\frac{4\sqrt{14}}{5}}{\frac{-31}{25}} = \frac{4\sqrt{14}}{5} \cdot \frac{-25}{31} = \frac{-20\sqrt{14}}{31}$