NAME ______________________ DATE ______________ PERIOD ________

7-1 Study Guide and Intervention
Parabolas

Analyze and Graph Parabolas A **parabola** is the locus of all points in a plane equidistant from a point called the **focus** and a line called the **directrix.** The standard form of the equation of a parabola that opens vertically is $(x-h)^2 = 4p(y-k)$. When p is negative, the parabola opens downward. When p is positive, it opens upward. The standard form of the equation of a parabola that opens horizontally is $(y-k)^2 = 4p(x-h)$. When p is negative, the parabola opens to the left. When p is positive, it opens to the right.

Example: For $(x-3)^2 = 12(y+4)$, identify the vertex, focus, axis of symmetry, and directrix. Then graph the parabola.

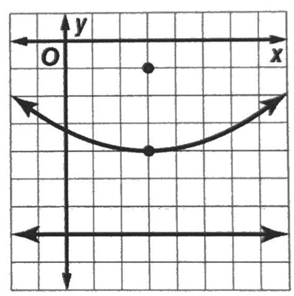
The equation is in standard form and the squared term is x, which means that the parabola opens vertically. Because $4p = 12$, $p = 3$ and the graph opens upward.

The equation is in the form $(x-h)^2 = 4p(y-k)$, so $h = 3$ and $k = -4$. Use the values of h, k, and p to determine the characteristics of the parabola.

vertex: (3, –4)	(h, k)	directrix:	$y = -7$	$y = k - p$
focus: (3, –1)	$(h, k + p)$	axis of symmetry:	$x = 3$	$x = h$

Graph the vertex, focus, axis, and directrix of the parabola. Then make a table of values to graph the general shape of the curve.

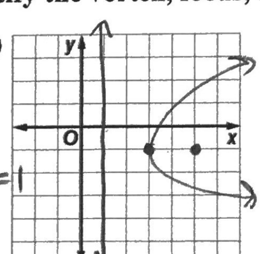
x	y
0	$-3\frac{1}{4}$
2	$-3\frac{11}{12}$
4	$-3\frac{11}{12}$
6	$-3\frac{1}{4}$



Exercises

For each equation, identify the vertex, focus, axis of symmetry, and directrix. Then graph the parabola.

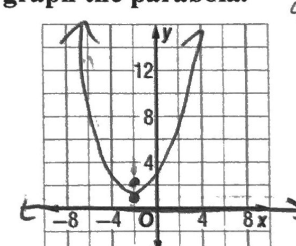
1. $(y + 1)^2 = 8(x-3)$



$8 = 4p$
$2 = p$

vertex: (3, −1)
aos: y = −1
directrix. x = 1
focus: (5, −1)

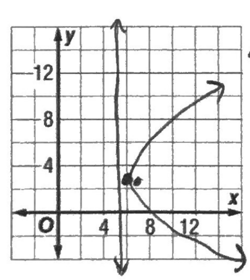
2. $(x+2)^2 = 4(y-1)$



$4p = 4$
$p = 1$

vertex: (−2, 1)
aos: x = −2
focus: (−2, 2)
directrix: y = 0

3. $(y-3)^2 = 2(x-6)$

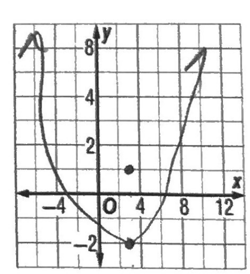


$2 = 4p$
$\frac{1}{2} = p$

vertex: (6, 3)
aos: y = 3
focus: (6.5, 3)
directrix: x = 5.5

4. $\frac{1}{12}(x-3)^2 = (y+2)$

$(x-3)^2 = 12(y+2)$



$4p = 12$
$p = 3$

vertex: (3, −2)
aos: x = 3
focus: (3, 1)
directrix: y = −5

NAME ______________________ DATE ______________ PERIOD ________

7-1 Study Guide and Intervention *(continued)*
Parabolas

Equations of Parabolas Specific characteristics can be used to determine the equation of a parabola.

Example: Write an equation for and graph a parabola with focus (–4, –3) and vertex (1, –3).

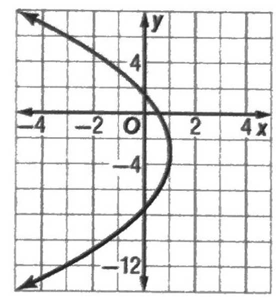
Because the focus and vertex share the same y-coordinate, the graph is horizontal. The focus is $(h + p, k)$, so the value of p is $-4 - 1$ or -5. Because p is negative, the graph opens to the left.

Write the equation for the parabola in standard form using the values of h, p, and k.

$(y - k)^2 = 4p\,(x - h)$	Standard form
$[y - (-3)]^2 = 4\,(-5)\,(x - 1)$	$p = -5$, $h = 1$, and $k = -3$
$(y + 3)^2 = -20\,(x - 1)$	Simplify.

The standard form of the equation is $(y + 3)^2 = -20\,(x - 1)$.

Graph the vertex, focus, and parabola.



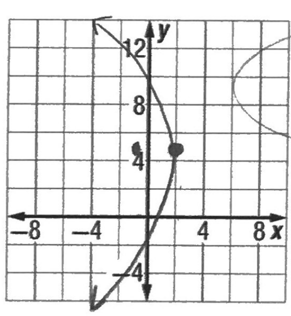
Exercises

Write an equation for and graph a parabola with the given characteristics.

1. focus (–1, 5) and vertex (2, 5)

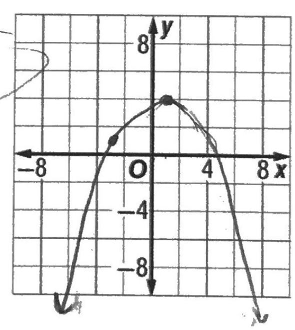
$p=3$

$(y-5)^2=4p\,(x-2)$



$(y-5)^2=-12(x-2)$

2. focus (1, 4) ; opens down; contains (–3, 1)



$(x-1)^2=4p(y-4)$

$(-3-1)^2=4p(1-4)$

$(-4)^2=4p(-3)$

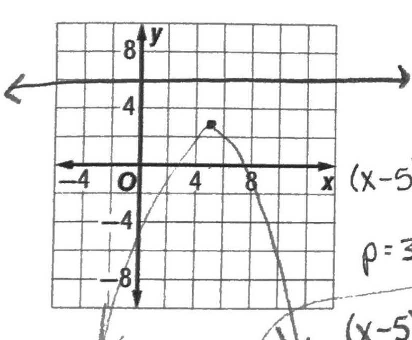
$16=-12p$

$-\frac{16}{12}=p$

$-\frac{4}{3}=p$

$(x-1)^2=-\frac{4}{3}(y-4)$

3. directrix $y = 6$; opens down; vertex (5, 3)



$(x-5)^2=4p(y-3)$

$p=3$

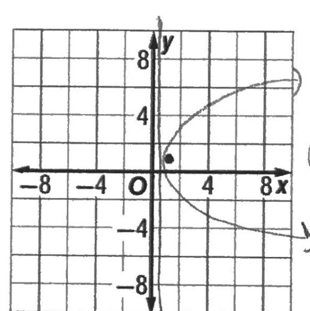
$(x-5)^2=-12(y-3)$

4. focus (1.5, 1); opens right; directrix $x = 0.5$

distance from focus & directrix = 1

$p=0.5$

$4p=2$



vertex: (1, 1)

$(y-1)^2=2(x-1)$

NAME ______________________ DATE ______________ PERIOD ________

7-1 Word Problem Practice
Parabolas

1. REFLECTOR The figure shows a parabolic reflecting mirror. A cross section of the mirror can be modeled by $x^2 = 16y$, where the values of x and y are measured in inches. Find the distance from the vertex to the focus of this mirror. $= p$

$(x-0)^2 = 16(y-0)$

$4p = 16$

$\boxed{p = 4\text{in}}$

2. T-SHIRTS The cheerleaders at the high school basketball game launch T-shirts into the stands after a victory. The launching device propels the shirts into the air at an initial velocity of 32 feet per second. A shirt's distance y in feet above the ground after x seconds can be modeled by $y = -16x^2 + 32x + 5$.

a. ~~Write the equation~~ in standard form.

$-16(x^2 - 2x) = y - 5$

$-16(x-1)^2 - (-16)(1) = y - 5$

$-16(x-1)^2 + 16 = y - 5$

$-16(x-1)^2 = y - 21$

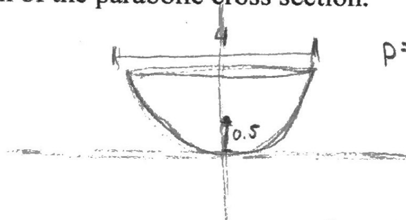
$(x-1)^2 = -\frac{1}{16}(y-21)$

b. What is the maximum height that a T-shirt reaches?

$(1, 21)$

$\boxed{21 \text{ feet}}$

3. FLASHLIGHT A flashlight contains a parabolic mirror with a bulb in the center as a light source and focus. If the width of the mirror is 4 inches at the top and the height to the focus is 0.5 inch, find an equation of the parabolic cross section.



$p: 0.5$

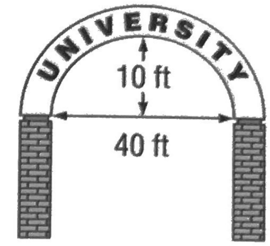
$(x-0)^2 = 4p(y-0)$

$(x-0)^2 = 2(y-0)$

$x^2 = 2y$

4. ARCHWAYS The entrance to a college campus has a parabolic arch above two columns as shown in the figure.

$p = 10$



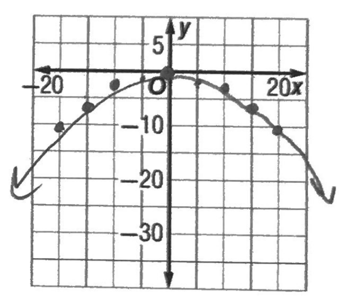
$(x-0)^2 = 4p(y-0)$

$(x-0)^2 = -40(y-0)$

a. Write an equation that models the parabola.

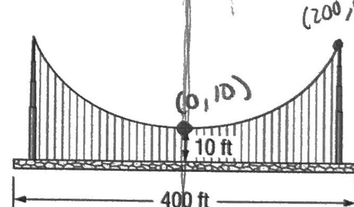
$\boxed{x^2 = -40y}$

b. Graph the equation.



x	y
-10	-2.5
-5	-0.625
0	0
5	-0.625
10	-2.5
-15	-5.625
15	-5.625
-20	-10
20	-10

5. BRIDGES The cable for a suspension bridge is in the shape of a parabola. The vertical supports are shown in the figure.



$(x-0)^2 = 4p(y-10)$

$(200-0)^2 = 4p(60-10)$

$40000 = 4p(50)$

$800 = 4p$

a. Write an equation for the parabolic cable.

$\boxed{x^2 = 800(y-10)}$

b. Find the length of a supporting wire that is 100 feet from the center.

$100^2 = 800(y-10)$

$12.5 = y - 10$

$\boxed{22.5 = y}$

NAME ______________________________ DATE ____________________ PERIOD ________

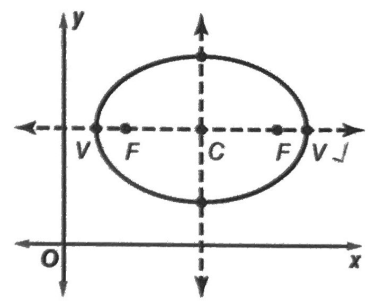
7-2 Study Guide and Intervention

Ellipses and Circles

Analyze and Graph Ellipses and Circles An **ellipse** is the locus of points in a plane such that the sum of the distances from two fixed points, called **foci,** is constant.

The standard form of the equation of an **ellipse** is

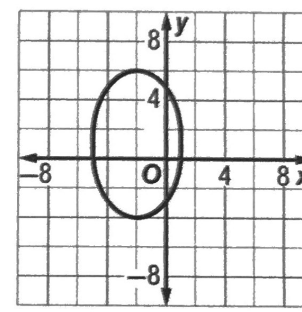
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ when the **major axis** is horizontal. In this case, a^2 is in the denominator of the x-term. The standard form is $\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$ when the major axis is vertical. In this case, a^2 is in the denominator of the y-term. In both cases, $c^2=a^2-b^2$.



Example: Graph the ellipse given by the equation $\frac{(y-1)^2}{25}+\frac{(x+2)^2}{9}=1$.

The equation is in standard form. Use the values of h, k, a, and b to determine the vertices and axes of the ellipse. Since $a^2>b^2$, $a^2=25$ and $b^2=9$, or $a=5$ and $b=3$. Since a^2 is the denominator of the y-term, the major axis is parallel to the y-axis.

orientation:	vertical	
center:	$(-2, 1)$	(h, k)
vertices:	$(-2, 6)$, $(-2, -4)$	$(h, k \pm a)$
co-vertices:	$(-5, 1)$, $(1, 1)$	$(h \pm b, k)$
major axis:	$x=-2$	$x=h$
minor axis:	$y=1$	$y=k$

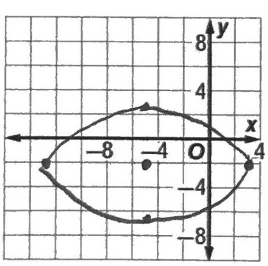


Exercises

Graph the ellipse given by each equation.

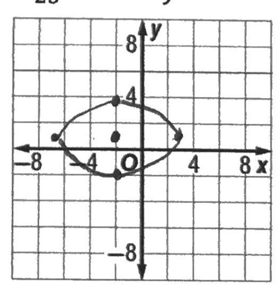
1. $\frac{(x+5)^2}{64}+\frac{(y+2)^2}{25}=1$

a=8
b=5
center: (-5, -2)



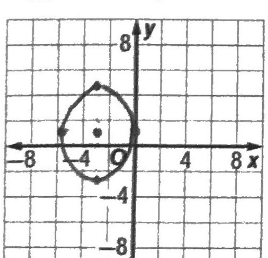
2. $\frac{(x+2)^2}{25}+\frac{(y+1)^2}{9}=1$

center: (-2, -1)
a=5
b=9



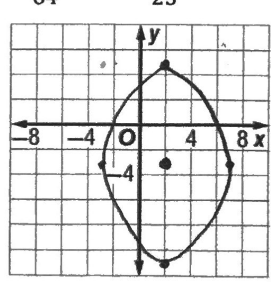
3. $\frac{(y-1)^2}{16}+\frac{(x+3)^2}{9}=1$

center: (-3, 1)
a=4
b=3



4. $\frac{(y+3)^2}{64}+\frac{(x-2)^2}{25}=1$

center: (2, -3)
a=8
b=5



NAME ____________________ DATE ____________ PERIOD ________

Study Guide and Intervention
Parabolas, Ellipses and Circles

Determine Types of Conic Sections If you are given the equation for a conic section, you can determine what type of conic is represented using the characteristics of the equation. The standard form of an equation for a circle with center (h, k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$.

Example: Write each equation in standard form. Identify the related conic.

a. $\mathbf{4x^2 + 9y^2 + 24x - 36y + 36 = 0}$

$4x^2 + 9y^2 + 24x - 36y + 36 = 0$ — Original equation

$4(x^2 + 6x + ?) + 9(y^2 - 4y + ?) = -36 + ? + ?$ — Complete the square.

$4(x^2 + 6x + 9) + 9(y^2 - 4y + 4) = -36 + 36 + 36$ — $\left(\frac{6}{2}\right)^2 = 9, \left(-\frac{4}{2}\right)^2 = 4$

$4(x+3)^2 + 9(y-2)^2 = 36$ — Factor.

$\frac{(x+3)^2}{9} + \frac{(y-2)^2}{4} = 1$ — Divide each side by 36.

Because the equation is of the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, the graph is an ellipse with center $(-3, 2)$.

b. $\mathbf{x^2 - 16x - 8y + 80 = 0}$

$x^2 - 16x - 8y + 80 = 0$ — Original equation

$(x^2 - 16x + ?) - 8y + 80 = 0$ — Complete the square.

$(x^2 - 16x + 64) - 8y + 80 - 64 = 0$ — $\left(\frac{16}{2}\right)^2 = 64$

$(x-8)^2 - 8(y-2) = 0$ — Factor.

$(x-8)^2 = 8(y-2)$ — Standard form

Because only one term is squared, the graph is a parabola with vertex $(8, 2)$.

Exercises

Write each equation in standard form. Identify the related conic. Sketch on graph paper and label all parts.

1. $y^2 + 2y + 6x^2 - 24x = 5$

$(y^2+2y)+6(x^2-4x)=5$

$(y+1)^2-1+6(x-2)^2-6(4)=5$

$(y+1)^2+6(x-2)^2=30$

$\boxed{\frac{(x-2)^2}{5}+\frac{(y+1)^2}{30}=1}$

$\boxed{\text{ellipse}}$

2. $y^2 + 2y + x^2 - 24x = 14$

$(y+1)^2-1+(x-12)^2-144=14$

$(y+1)^2+(x-12)^2=159$

$\boxed{\frac{(x+12)^2}{159}+\frac{(y+1)^2}{159}=1}$

$\boxed{\text{circle}}$

3. $4x - 8 + y^2 + 4y = 0$

$y^2+4y=-4x+8$

$(y+2)^2-4=-4x+8$

$(y+2)^2=-4x+12$

$\boxed{(y+2)^2=-4(x-3)}$

$\boxed{\text{parabola}}$

4. $x^2 + 4x + y^2 - 2y - 49 = 0$

$x^2+4x+y^2-2y=49$

$(x+2)^2-4+(y-1)^2-1=49$

$(x+2)^2+(y-1)^2=54$

$\boxed{\frac{(x+2)^2}{54}+\frac{(y-1)^2}{54}=1}$

$\boxed{\text{circle}}$

5. $4x^2 + 8x + 5y^2 - 30y - 11 = 0$

$4(x^2+2x)+5(y^2-6y)=11$

$4(x+1)^2-4(1)+5(y-3)^2-5(9)=11$

$4(x+1)^2+5(y-3)^2=60$

$\boxed{\frac{(x+1)^2}{15}+\frac{(y-3)^2}{12}=1}$

$\boxed{\text{ellipse}}$

6. $6x^2 + 24x + 2y - 10 = 0$

$6(x^2+4x)=-2y+10$

$6(x+2)^2-6(4)=-2y+10$

$6(x+2)^2=-2y+34$

$6(x+2)^2=-2(y-17)$

$\boxed{(x+2)^2=-\frac{1}{3}(y-17)}$

$\boxed{\text{parabola}}$

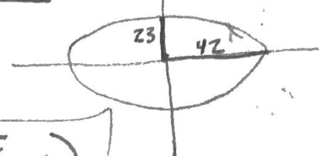
NAME ______________________ DATE ______________ PERIOD ________

7-2 Word Problem Practice
Ellipses and Circles

1. WHISPERING GALLERY A whispering gallery at a museum is in the shape of an ellipse. The room is 84 feet long and 46 feet wide.

a. Write an equation modeling the shape of the room. Assume that it is centered at the origin and that the major axis is horizontal.

$$\boxed{\frac{x^2}{1764}+\frac{y^2}{529}=1}$$



b. Find the location of the foci.

$c^2=a^2-b^2$

$c=\sqrt{1235}$

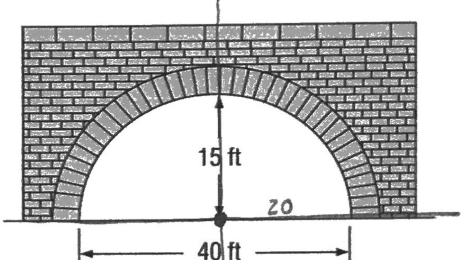
$\boxed{\text{foci: } (\pm\sqrt{1235}, 0)}$

2. SIGNS A sign is in the shape of an ellipse. The eccentricity is 0.60 and the length is 48 inches.

a. Write an equation for the ellipse if the center of the sign is at the origin and the major axis is horizontal.

b. What is the maximum height of the sign?

3. TUNNEL The entrance to a tunnel is in the shape of half an ellipse as shown in the figure.



a. Write an equation that models the ellipse.

$$\boxed{\frac{x^2}{400}+\frac{y^2}{225}=1}$$

b. Find the height of the tunnel 10 feet from the center.

$\frac{10^2}{400}+\frac{y^2}{225}=1$

$\frac{1}{4}+\frac{y^2}{225}=1$

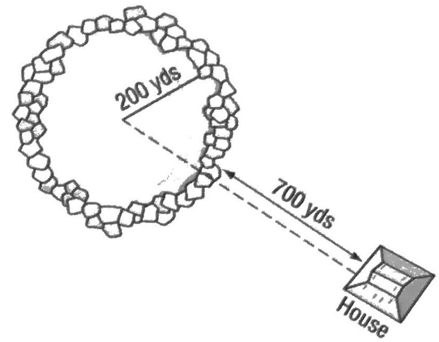
$\frac{y^2}{225}=\frac{3}{4}$

$4y^2=675$

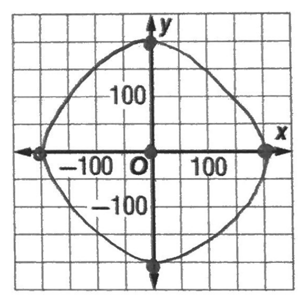
$y^2=\frac{675}{4}$

$\boxed{y\approx 13 \text{ ft}}$

4. RETENTION POND A circular retention pond is getting larger by overflowing and flooding the nearby land at a rate that increases the radius 100 yards per day, as shown below.



a. Graph the circle that represents the water, and find the distance from the center of the pond to the house.



distance from center of pond → house = 900 yds

b. If the pond continues to overflow at the same rate, how many days will it take for the water to reach the house?

day	radius
0	200
1	300
2	400
3	500
4	600

day	radius
5	700
6	800
7	900

c. Write an equation for the circle of water at the current time and an equation for the circle when the water reaches the house.

$x^2+y^2=200^2$

$x^2+y^2=40{,}000$

$x^2+y^2=900^2$

$x^2+y^2=810{,}000$

NAME ______________________ DATE ______________ PERIOD ________

7-3 Study Guide and Intervention
Hyperbolas

Analyze and Graph Hyperbolas A **hyperbola** is the locus of all points in a plane such that the difference of their distances from two foci is constant. The standard form of the equation of a **hyperbola** is

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ when the **transverse axis** is horizontal, and

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ when the transverse axis is vertical. In both cases, $a^2 + b^2 = c^2$.

Example: Graph the hyperbola given by the equation $\frac{y^2}{16} - \frac{x^2}{4} = 1$.

The equation is in standard form. Both h and k are 0, so the center is at the origin. Because the x-term is subtracted, the transverse axis is vertical. Use the values of a, b, and c to determine the vertices and foci of the hyperbola.

Because $a^2 = 16$ and $b^2 = 4$, $a = 4$ and $b = 2$. Use the values of a and b to find the value of c.

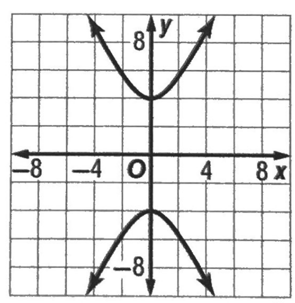
$c^2 = a^2 + b^2$	Equation relating a, b, and c
$c^2 = 4^2 + 2^2$	$a = 4$ and $b = 2$
$c = \sqrt{20}$ or about 4.47	Simplify.

Determine the characteristics of the hyperbola.

center: $(0, 0)$	(h, k)	foci:	$(0, \sqrt{20})$, $(0, -\sqrt{20})$	$(h, k \pm c)$
vertices: $(0, 4)$, $(0, -4)$	$(h, k \pm a)$	asymptotes:	$y = 2x$, $y = -2x$	$y - k = \pm\frac{a}{b}(x - h)$

Make a table of values to sketch the hyperbola.

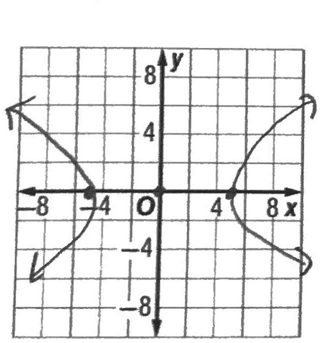
x	y
−2	− 5.65, 5.65
−1	− 4.5, 4.5
0	− 4, 4
1	− 4.5, 4.5
2	− 5.65, 5.65



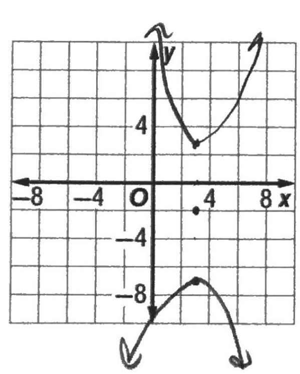
Exercises

Graph the hyperbola given by each equation.

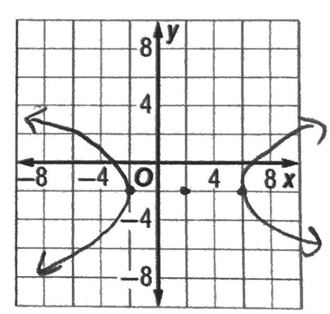
1. $\frac{x^2}{25} - \frac{y^2}{36} = 1$



2. $\frac{(y-3)^2}{25} - \frac{(x+2)^2}{9} = 1$



3. $\frac{(x-1)^2}{16} - \frac{(y+2)^2}{36} = 1$



NAME ______________________ DATE ______________ PERIOD ________

7-3 Study Guide and Intervention *(continued)*

Hyperbolas

Identify Conic Sections You can determine the type of conic when the equation for the conic is in general form, $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. The discriminant, or $B^2 - 4AC$, can be used to identify a conic when the equation is in general form.

Discriminant	Conic Section
less than 0; $B = 0$ and $A = C$	circle
less than 0; $B \neq 0$ or $A \neq C$	ellipse
equal to 0	parabola
greater than 0	hyperbola

Example: Use the discriminant to identify each conic section.

a. $\mathbf{2x^2 + 6y^2 - 8x + 12y - 2 = 0}$

A is 2, B is 0, and C is 6. Find the discriminant.

$B^2 - 4AC = 0^2 - 4(2)(6)$ or -48

The discriminant is less than 0, so the conic must be either a circle or an ellipse. Because $A \neq C$, the conic is an ellipse.

also these will be more obvious on your test. ✓

b. $\mathbf{5x^2 + 8xy - 2y^2 + 4x - 3y + 10 = 0}$

A is 5, B is 8, and C is -2. Find the discriminant.

$B^2 - 4AC = 8^2 - 4(5)(-2)$ or 104.

The discriminant is greater than 0, so the conic is a hyperbola.

c. $\mathbf{12x^2 + 12xy + 3y^2 - 7x + 2y - 6 = 0}$

A is 12, B is 12, and C is 3. Find the discriminant.

$B^2 - 4AC = 12^2 - 4(12)(3)$ or 0

The discriminant is 0, so the conic is a parabola.

* Don't need to use discriminant to be able to tell.

Exercises

Use the discriminant to identify each conic section.

1. $4x^2 + 4y^2 - 2x - 9y + 1 = 0$

$4x^2 - 2x + 4y^2 - 9y = -1$

$4(x^2 - \frac{1}{2}x) + 4(y^2 - \frac{9}{4}y) = -1$

$-1 + 4(\frac{1}{16}) + 4(\frac{36}{16}) = +$ circle

2. $10x^2 + 6y^2 - x + 8y + 1 = 0$

3. $-2x^2 + 6xy + y^2 - 4x - 5y + 2 = 0$

4. $x^2 + 6xy + y^2 - 2x + 1 = 0$

5. $5x^2 + 2xy + 4y^2 + x + 2y + 17 = 0$

6. $x^2 + 2xy + y^2 + x + 10 = 0$

$(x + y)^2 + x + 10 = 0$

7. $25x^2 + 100x - 54y = -200$

parabola

8. $16x^2 + 100x - 54y^2 = -100$

$16(x^2 + \frac{25}{4}x) - 54y^2 = -100$

hyperbola

only x^2 or y^2 = parabola

$+x^2$ and $+y^2$ same denoms. = circle

different denom. = ellipse

$-x^2$ or $-y^2$ = hyperbola

NAME ______________________ DATE ______________ PERIOD ________

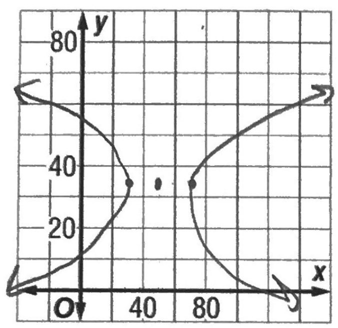
7-3 Word Problem Practice
Hyperbolas

1. EARTHQUAKES The epicenter of an earthquake lies on a branch of the hyperbola represented by $\frac{(x-50)^2}{1600} - \frac{(y-35)^2}{2500} = 1$, where the seismographs are located at the foci.

a. Graph the hyperbola.

center: $(50, 35)$

$a = 40$



b. Find the locations of the seismographs.

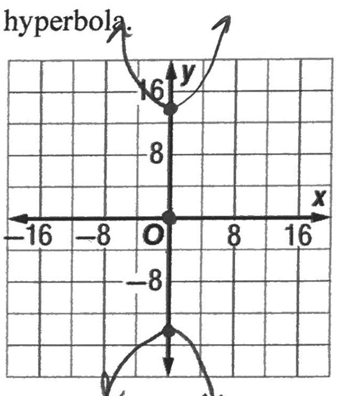
$c^2 = a^2 + b^2$

$c = 10\sqrt{41}$

$\boxed{(30 \pm 10\sqrt{41}, 35)}$

2. SHADOWS A lamp projects light onto a wall in the shape of a hyperbola. The edge of the light can be modeled by $\frac{y^2}{196} - \frac{x^2}{121} = 1$.

a. Graph the hyperbola.



$c^2 = a^2 + b^2$

$14^2 = 12^2 + b^2$

$52 = b$

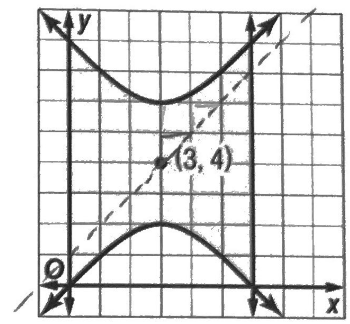
b. Write the equations of the asymptotes.

$(y-k) = \pm \frac{a}{b}(x-h)$

$\boxed{y = \pm \frac{196}{121} x}$

~~c. Find the eccentricity.~~

3. PARKS A grassy play area is in the shape of a hyperbola, as shown.



$y - k = \pm \frac{a}{b}(x-h)$

$\frac{a}{b} = 1$

$a = b$

a. Write an equation that models the curved sides of the play area.

$\boxed{\frac{(y-4)^2}{4} - \frac{(x-3)^2}{4} = 1}$

b. If each unit on the coordinate plane represents 3 feet, what is the narrowest vertical width of the play area?

$\boxed{4(3) = 12 \text{ ft}}$

4. SHADOWS The path of the shadow cast by the tip of a sundial is usually a hyperbola.

a. Write two equations of the hyperbola in standard form if the center is at the origin, given that the path contains a transverse axis of 24 millimeters with one focus 14 millimeters from the center.

$a = 12$

c ✓ $\frac{x^2}{144} - \frac{y^2}{2704} = 1$ or $\frac{y^2}{144} - \frac{x^2}{2704} = 1$

b. Graph one hyperbola.