Day 1

Wednesday, September 13, 2017
12:35 PM

Plan:
1. Power Function - Jigsaw
2. Power Function Regression - Notes
3. Solving Radical Equations - Notes

Jigsaw:

All people assigned problem A get together and answer your questions. Then go back to your original group (1 A, 1 B, 1 C, and 1 D) and teach your group about your problem.

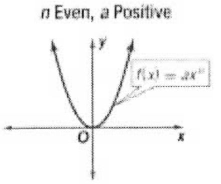
All people assigned problem B get together and answer your questions. Then go back to your original group (1 A, 1 B, 1 C, and 1 D) and teach your group about your problem.

...

KeyConcept Monomial Functions

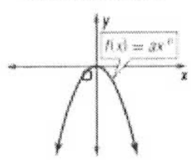
Let f be the power function $f(x) = ax^n$, where n is a positive integer.

n Even, a Positive



Domain: $(-\infty, \infty)$ **Range:** $[0, \infty)$
x- and y-Intercept: 0
Continuity: continuous for $x \in \mathbb{R}$
Symmetry: y-axis **Minimum:** $(0, 0)$
Decreasing: $(-\infty, 0)$ **Increasing:** $(0, \infty)$
End behavior: $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = \infty$

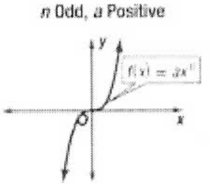
n Even, a Negative



Domain: $(-\infty, \infty)$ **Range:** $(-\infty, 0]$
x- and y-Intercept: 0
Continuity: continuous for $x \in \mathbb{R}$
Symmetry: y-axis **Maximum:** $(0, 0)$
Decreasing: $(0, \infty)$ **Increasing:** $(-\infty, 0)$
End behavior: $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = -\infty$

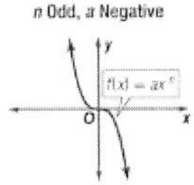
all even functions will have end behavior in same direction.

n Odd, a Positive



Domain and Range: $(-\infty, \infty)$
x- and y-Intercept: 0
Continuity: continuous on $(-\infty, \infty)$
Symmetry: origin
Extrema: none **Increasing:** $(-\infty, \infty)$
End Behavior: $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$

n Odd, a Negative

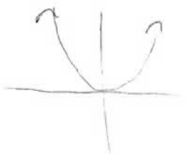


Domain and Range: $(-\infty, \infty)$
x- and y-Intercept: 0
Continuity: continuous for $x \in \mathbb{R}$
Symmetry: origin
Extrema: none **Decreasing:** $(-\infty, \infty)$
End Behavior: $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = -\infty$

all odd functions will have end behavior in opposite directions

Example 1: Graph and analyze the functions. Describe the domain, range, intercepts, end behavior, continuity, and where the function is increasing or decreasing.

A. $f(x) = \frac{1}{2}x^6$



domain: $\mathbb{R}$

range: $[0, \infty)$

x-intercepts: $0 = \frac{1}{2}x^6$

$0 = x^6$

$0 = x$ $(0,0)$

y-intercepts: $y = \frac{1}{2}(0)^6$

$y = 0$ $(0,0)$ > origin

end behavior: $\lim_{x \to -\infty} f(x) = \infty$

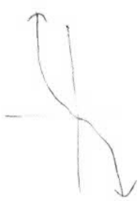
$\lim_{x \to \infty} f(x) = \infty$

continuity: continuous everywhere

increasing: $(0, \infty)$

decreasing: $(-\infty, 0)$

B. $f(x) = -x^5$



domain: $\mathbb{R}$

range: $\mathbb{R}$

x-int: $0 = -x^5$

$0 = x^5$

$0 = x$

y-int: $y = -(0)^5$ $(0,0)$

$y = 0$

end behavior: $\lim_{x \to -\infty} f(x) = \infty$

$\lim_{x \to \infty} f(x) = -\infty$

continuity: continuous everywhere

increasing: never

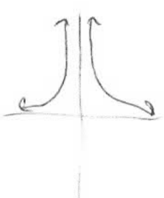
decreasing: $(-\infty, \infty)$

Negative Exponents & Rational Functions

- Remember:
  - $x^{-1} = \frac{1}{x}$
  - $x^{-2} = \frac{1}{x^2}$
  - $x^{-3} = \frac{1}{x^3}$
  - etc.
- Undefined at $x = 0$
- Graphs with negative exponents will contain discontinuities

Example 2: Graph and analyze. Describe the domain, range, intercepts, end behavior, continuity, and where the function is increasing or decreasing.

A. $f(x) = 2x^{-4}$ $\left(\frac{2}{x^4}\right)$



domain: $x \neq 0$

range: $y > 0$

intercepts: none

end behavior: $\lim_{x \to -\infty} f(x) = 0$

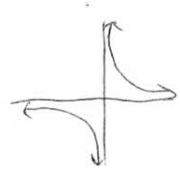
$\lim_{x \to \infty} f(x) = 0$

continuity: infinite discontinuity @ $x = 0$

increasing: $(-\infty, 0)$

decreasing: $(0, \infty)$

B. $f(x) = 3x^{-5}$ $\left(\frac{3}{x^5}\right)$



domain: $x \neq 0$

range: $y \neq 0$

intercepts: none

end behavior: $\lim_{x \to -\infty} f(x) = 0$

$\lim_{x \to \infty} f(x) = 0$

continuity: infinite discontinuity @ $x = 0$

increasing: never

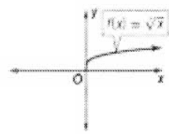
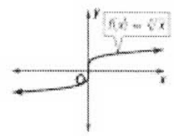
decreasing: $(-\infty, 0) \cup (0, \infty)$

Radical Functions

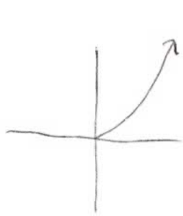
- $x^{\frac{p}{n}} = \sqrt[n]{x^p}$
- Examples:
  - $f(x) = 3\sqrt{5x^3}$
  - $f(x) = -5\sqrt[3]{x^4 + 3x^2 - 1}$
  - $f(x) = \sqrt[4]{x + 12} + \frac{1}{2}x - 7$

KeyConcept Radical Functions

Let f be the radical function $f(x) = \sqrt[n]{x}$ where n is a positive integer.

n Even	n Odd
	
Domain and Range: $[0, \infty)$	**Domain and Range:** $(-\infty, \infty)$
x- and y-Intercept: 0	**x- and y-Intercept:** 0
Continuity: continuous on $[0, \infty)$	**Continuity:** continuous on $(-\infty, \infty)$
Symmetry: none **Increasing:** $(0, \infty)$	**Symmetry:** origin **Increasing:** $(-\infty, \infty)$
Extrema: absolute minimum at $(0, 0)$	**Extrema:** none
End Behavior: $\lim_{x \to \infty} f(x) = \infty$	**End Behavior:** $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$

Example 5: Graph and analyze the functions. Describe the domain, range, intercepts, end behavior, continuity, and where the function is increasing or decreasing.



A. $f(x) = 5\sqrt{2x^3}$ $\left(5(2x^3)^{1/2}\right)$

domain: $x \geq 0$

range: $y \geq 0$

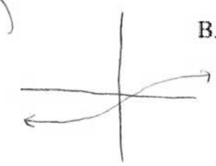
x-y intercepts: $(0,0)$

end behavior: $\lim_{x \to \infty} f(x) = \infty$

continuity: continuous on $[0, \infty)$

increasing: $(0, \infty)$

decreasing: never



B. $f(x) = \frac{1}{2}\sqrt[5]{3x - 4}$ $\left(\frac{1}{2}(3x-4)^{1/5}\right)$

domain: $\mathbb{R}$

range: $\mathbb{R}$

x-int: $0 = \frac{1}{2}\sqrt[5]{3x-4}$

$0 = \sqrt[5]{3x-4}$

$0 = 3x - 4$

$\frac{4}{3} = x$ $\left(\frac{4}{3}, 0\right)$

y-int: $y = \frac{1}{2}\sqrt[5]{3(0)-4}$

$y = \frac{1}{2}\sqrt[5]{-4}$

$y = -0.6597$

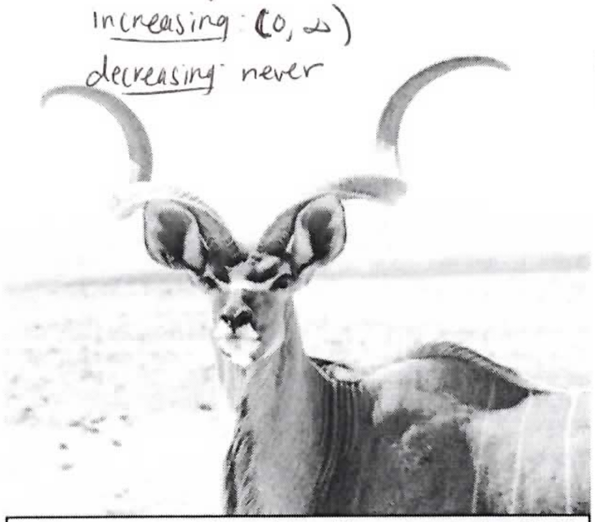
$(0, -0.6597)$

end behavior: $\lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = \infty$

increasing: $(-\infty, \infty)$

decreasing: never



Antelopes are native to Africa and Asia. They range in size from 12" (30 cm. at the shoulder) pygmy antelopes to giant elands, which are over 6 feet tall (180 cm) at the shoulder. Most antelopes are between 3 to 4 feet tall (90-120 cm) at the shoulder. The horns of antelopes, unlike the antlers of deer, are un-branched, are made of a shell with a bony core, and are not

Data: The data below represents the length and mid-shaft diameters of the humerus bones of African Antelopes.

Diameter (mm)	Length (mm)
17.6	159.9
26.0	206.9
31.9	236.8
38.9	269.9
45.8	300.6
51.2	323.6
58.1	351.7
64.7	377.6

cm. at the shoulder) pygmy antelopes to giant elands, which are over 6 feet tall (180 cm) at the shoulder. Most antelopes are between 3 to 4 feet tall (90-120 cm) at the shoulder. The horns of antelopes, unlike the antlers of deer, are un-branched, are made of a shell with a bony core, and are not shed. The majority of antelopes reside in Africa.

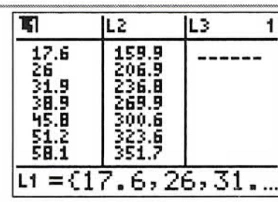
The kudus antelope, shown here, relies on thickets for protection using his brown and striped coat as camouflage.

58.1	351.7
64.7	377.6
66.7	384.1
80.8	437.2
82.9	444.7

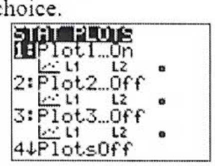
Task: Express answers to the *nearest tenth.*

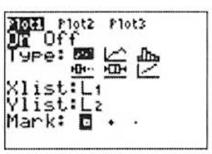
a.) Prepare a scatter plot of the data.
b.) Determine a power regression model equation to represent this data.
c.) Graph the new equation.
d.) Decide whether the new equation is a "good fit" to represent this data.
e.) *Extrapolate data:* What length will correspond to a diameter of 84 mm?
f.) *Interpolate data:* What length will correspond to a diameter of 47 mm?
g.) What mid-shaft diameter will correspond to a length of 305.7 mm?

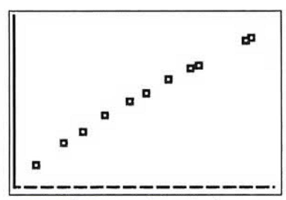
Step 1. Enter the data into the lists.
For basic entry of data, see Basic Commands.



Step 2. Create a scatter plot of the data.
Go to STATPLOT (2nd Y=) and choose the first plot. Turn the plot ON, set the icon to Scatter Plot (the first one), set Xlist to L1 and Ylist to L2 (assuming that is where you stored the data), and select a Mark of your choice.






(answer to part a)

Step 3. Choose the Power Regression Model.

Press STAT, arrow right to CALC, and arrow down to A: PwrReg. Hit ENTER. When PwrReg appears on the home screen, type the parameters L1, L2, Y1. The Y1 will put the equation into Y= for you. (Y_1 comes from VARS → YVARS, #Function, Y_1)

```
EDIT CALC TESTS
6↑CubicReg
7:QuartReg
8:LinReg(a+bx)
9:LnReg
0:ExpReg
A:PwrReg
B↓Logistic
```

```
PwrReg L1,L2,Y1
```

```
PwrReg
 y=a*x^b
 a=24.12989312
 b=.65949782
 r²=.9999925076
 r=.9999962538
```

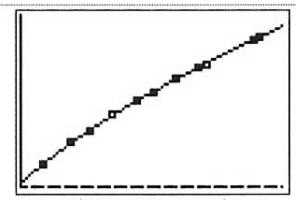
The power regression equation is

$$y = 24.1x^{0.7}$$

(answer to part b)

Step 4. Graph the Power Regression Equation from Y1.

ZOOM #9 ZoomStat to see the graph.



(answer to part c)

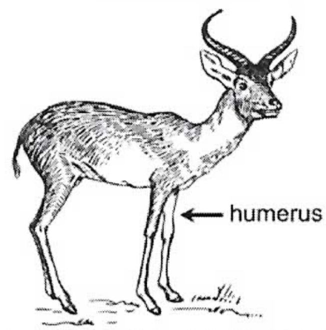
Step 5. Is this model a "good fit"?

The correlation coefficient, r, is .9999937121 which indicates a very strong correlation since it is close to 1.

The coefficient of determination, r^2, is .99999250766 which means that 99% of the total variation in y can be explained by the relationship between x and y.

Yes, it is a very "good fit".

(answer to part d)



Step 6. Extrapolate: *(beyond the data set)*

Go to TBLSET (above WINDOW) and set the TblStart to 84. Notice how setting the increment (the deltaTbl = 0.1) displays certain values to two decimal places. The calculator is trying to tell you that this value has been "rounded" to the nearest hundredth and should not be now rounded to the nearest tenth.
Always check the "full" Y1 value, as seen at the bottom of the screen, before rounding.
(See an alternate method in Step 7.)

```
TABLE SETUP
 TblStart=84
 ΔTbl=.1
Indpnt: Auto Ask
Depend: Auto Ask
```

X	Y_1
84	448.35
84.1	448.7
84.2	449.05
84.3	449.4
84.4	449.75
84.5	450.1
84.6	450.46

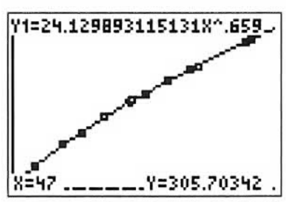
Y_1=448.346095927

(answer to part e - the length will be 448.3 mm)

Step 7. Interpolate:

(within the data set)

From the graph screen, hit TRACE, arrow up to obtain the power equation, type 47, hit ENTER, and the answer will appear at the bottom of the screen.



(answer to part f - length will be 305.7 mm)

Step 8. What mid-shaft diameter will correspond to a length of 305.7 mm?

In reference to the equation, go to TABLE (above GRAPH) and arrow down until you find a Y1 value equal to (or close to) the desired value. The answer will appear in the X column. 47 mm.

X	Y_1
46.8	304.84
46.9	305.27
47	305.7
47.1	306.13
47.2	306.56
47.3	306.99
47.4	307.42

X=47

Solve radical equations.

Example 1: $\left(\sqrt{2x-3}\right)^2 = (5)^2$

$2x-3=25$

$2x=28$

$x=14$

Example 2: $\sqrt[3]{(x-5)^2}+14=50$

$\left(\sqrt[3]{(x-5)^2}\right)^3=(36)^3$

$(x-5)^2=46656$

$x-5=\pm 216$

$x-5=216$ $\qquad$ $x-5=-216$

$x=221$ $\qquad$ $x=-211$

Example 3: $2x=\sqrt{100-12x}-2$

$(2x+2)^2=\left(\sqrt{100-12x}\right)^2$

$4x^2+8x+4=100-12x$

$4x^2+20x-96=0$

$x^2+5x-24=0$

$(x+8)(x-3)=0$

$x=-8$ $\quad$ $x=3$