Day 5

Thursday, August 31, 2017
3:12 AM

1. Warm-up/Homework
2. Grand Teton Range Activity
3. Notes on 1.4
4. Assignment **Pg 41-42** 7,13,46-49, 51, 54-59, 70, 72

5-Minute Check (over Lesson 1-3) — Use with Lesson **1-4**

Determine whether each function is continuous at the given *x*-value.

1. $y = x^2 + x - 5;\ x = 7$

2. $y = \frac{2x-3}{x-4};\ x = 4$

3. $f(x) = \begin{cases} x + 6 & \text{if } x \geq 2 \\ x^2 + 4 & \text{if } x < 2 \end{cases};\ x = 2$

4. Describe the end behavior of $f(x) = -6x^4 + 3x^3 - 17x^2 - 5x + 12$.

$\lim_{x \to \pm\infty} f(x) = \infty$

Standardized Test Practice

5. Determine between which consecutive integers the real zeros of $f(x) = x^3 + x^2 - 2x + 5$ are located on the interval $[-4, 4]$.

A $[-2, -1]$　　**C** $[0, 1]$

B $[-3, -2]$　　**D** $[-4, -3]$

ANSWERS

1. yes　　2. no　　3. yes
4. As $x \to \infty$, $f(x) \to -\infty$ and as $x \to -\infty$, $f(x) \to -\infty$.
5. B

Chapter 1　　Glencoe Precalculus

1. $f(7) = (7)^2 + (7) - 5 = 51$ ✓

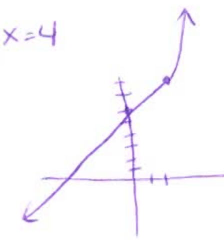
$\lim_{x \to 7^-} f(x) = 51$

$\lim_{x \to 7^+} f(x) = 51$

2. no, discontinuous at $x = 4$

3. $f(2) = (2) + 6 = 8$

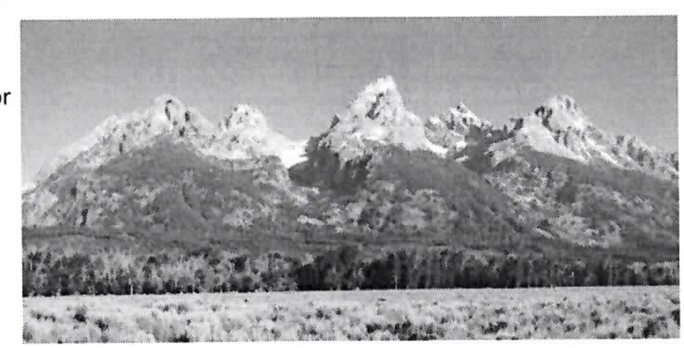
yes, continuous at $x = 2$



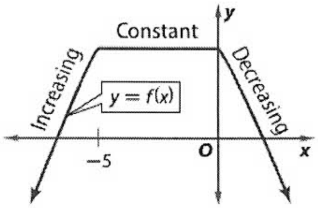
-4	-3	-2	-1	0	1	2	3	4
-35	-7	3	7	5	5	13	.	...

Today's Objectives

- Determine intervals on which functions are increasing, constant, or decreasing, and determine maxima and minima of functions.
- Determine the average rate of change of a function.



If you walk along the ridge of the Teton Range in Wyoming, where would you increase and decrease in altitude and where would you reach maximum and minimum points?



Consider the graph of $f(x)$ shown. As you move from left to right, $f(x)$ is...

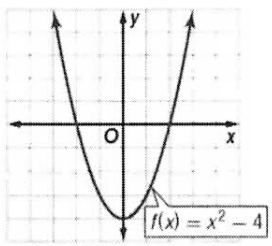
Increasing on $(-\infty, -5)$ > pick one to have the closed bracket on 5 Doesn't matter which one.

Constant on $[-5, 0]$

Decreasing on $(0, \infty)$ > pick one to have the closed bracket on 0.

EXAMPLE 1 Analyze Increasing and Decreasing Behavior

A. Use the graph of the function $f(x) = x^2 - 4$ to estimate intervals to the nearest 0.5 unit on which the function is increasing, decreasing, or constant. Support the answer numerically.



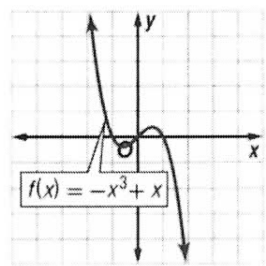
increasing: $(0, \infty)$

decreasing: $(-\infty, 0]$

constant: $x = 0$

EXAMPLE 1 Analyze Increasing and Decreasing Behavior

B. Use the graph of the function $f(x) = -x^3 + x$ to estimate intervals to the nearest 0.5 unit on which the function is increasing, decreasing, or constant. Support the answer numerically.

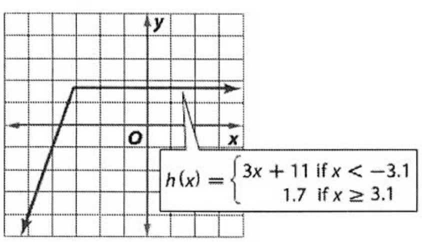


decreasing: $(-\infty, -0.5) \cup (0.5, \infty)$

increasing: $(-0.5, 0.5)$

constant: $x = -0.5$, $x = 0.5$

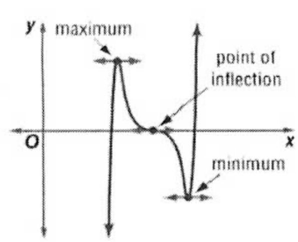
You Try:



increasing: $(-\infty, -3.1)$

constant: $[-3.1, \infty)$

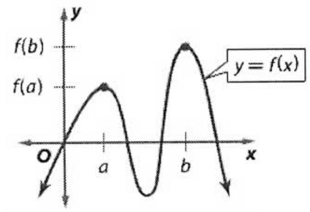
Extrema are critical points at which a function changes its increasing or decreasing behavior. At these points the function has a maximum or a minimum value.



All maximum values are **relative or local maximums** on a given interval.

An **Absolute maximum** is the largest value of the function over the functions domain. **Note:** If a function's end behavior approaches infinity, it will not have an absolute maximum.

Model

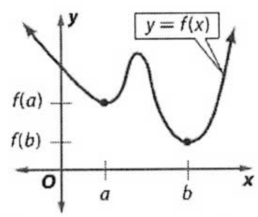


$f(a)$ is a relative maximum of f.
$f(b)$ is the absolute maximum of f.

All minimum values are **relative or local minimums** on a given interval.

An **Absolute minimum** is the smallest value of the function over the functions domain. **Note:** If a function's end behavior approaches negative infinity, it will not have an absolute minimum.

Model

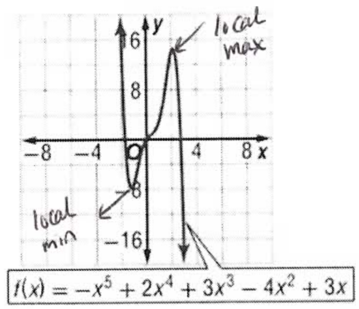


$f(a)$ is a relative minimum of f.
$f(b)$ is the absolute minimum of f.

EXAMPLE 2 Estimate and Identify Extrema of a Function

local = relative

Estimate and classify the extrema to the nearest 0.5 unit for the graph of $f(x)$. Support the answers numerically.



no absolute min or max

local min (−1, −8)

local max (2, 14)

Finding max and/or min:

FUEL ECONOMY Advertisements for a new car claim that a tank of gas will take a driver and three passengers about 360 miles. After researching on the Internet, you find the function for miles per tank of gas for the car is $f(x) = -0.025x^2 + 3.5x + 240$, where x is the speed in miles per hour. What speed optimizes the distance the car can travel on a tank of gas? How far will the car travel at that optimum speed?

max or min for quadratic: $x = \frac{-b}{2a}$

$x = \frac{-3.5}{2(-0.025)} = 70$ x=speed → optimum speed = 70 mph

$y = -0.025(70)^2 + 3.5(70) + 240$
$= 362.5$ y=miles ⇒ car can travel 362.5 miles.

KeyConcept Average Rate of Change

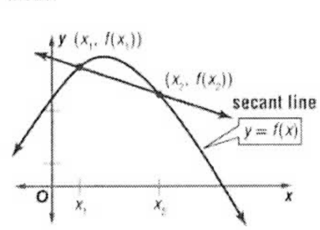
Words The **average rate of change** between any two points on the graph of f is the slope of the line through those points.

Geometry The line through two points on a curve is called a **secant line**. The slope of the secant line is denoted m_{sec}.

Symbols The average rate of change on the interval $[x_1, x_2]$ is

$$m_{sec} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

Model



EXAMPLE 5 Find Average Rates of Change

A. Find the average rate of change of $f(x) = -2x^2 + 4x + 6$ on the interval [–3, –1].

$\frac{f(-1) - f(-3)}{-1 - -3} = \frac{0 - -24}{2} = \frac{24}{2} = 12$

B. Find the average rate of change of $f(x) = -2x^2 + 4x + 6$ on the interval [2, 5].

$\frac{f(5) - f(2)}{5 - 2} = \frac{-24 - 6}{3} = \frac{-30}{3} = -10$

Average rate of change has many real-world applications. One common application involves the average speed of an object traveling over a distance d or from a height h in a given period of time t. Because speed is distance traveled per unit time, the average speed of an object cannot be negative.

Real-World Example 6 Find Average Speed

PHYSICS The height of an object that is thrown straight up from a height of 4 feet above ground is given by $h(t) = -16t^2 + 30t + 4$, where t is the time in seconds after the object is thrown. Find and interpret the average speed of the object from 1.25 to 1.75 seconds.

$\frac{f(1.75) - f(1.25)}{1.75 - 1.25} = \frac{7.5 - 16.5}{0.5} = \frac{-9}{0.5} = -18$ ft/s

The object is dropping back ~~t~~owards the ground at 18 ft/s

Also, what is the maximum height the object reached?

$$t = -\frac{b}{2a}$$

$$t = -\frac{30}{2(-16)} = 0.9375 \text{ sec}$$

$$h = -16(0.9375)^2 + 30(0.9375) + 4$$

$$= 18.0625 \text{ ft.}$$