Day 4

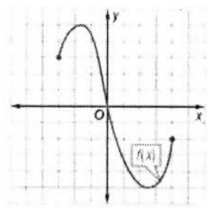
Wednesday, August 30, 2017
7:33 PM

I. Warm-up / Homework Solutions
II. Explore continuity with road/bridge analogy
III. Notes on 1.3
IV. Assignment **Pg 31** # 42,43, 45, 47,49, 56, 59, 61-66

5-Minute Check (over Lesson 1-2)

Use with Lesson **1-3**

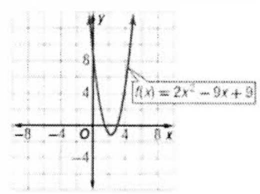
1. Use the graph of $f(x)$ to find the domain and range of the function.



D: $[-3, 4]$

R: $[-5, 5]$

2. Use the graph of $f(x)$ to find the y-intercept and zeros. Then find these values algebraically.



y-int = 9

zeros: ~~1.75~~ 1.5, 3?

$0 = 2x^2 - 9x + 9$

$x = \dfrac{9 \pm \sqrt{81 - 4(2)(9)}}{4}$

$x = \dfrac{9 \pm \sqrt{9}}{4}$

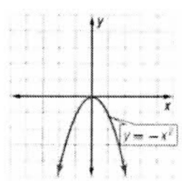
$x = \dfrac{9 \pm 3}{4} = 3, \dfrac{3}{2}$

$y = 2(0)^2 - 9(0) + 9$

$= 9$

Standardized Test Practice

3. Use the graph of $y = -x^2$ to test for symmetry with respect to the x-axis, y-axis, and the origin.



A y-axis symmetry
B x-axis symmetry
C origin symmetry
D x- and y-axis symmetry

ANSWERS
1. D = $[-3, 4]$, R = $[-5, 5]$
2. y-intercept = 9, zeros: 1.5 and 3
3. A

Chapter 1 Glencoe Precalculus

Today's Objectives:

- Use limits to determine the continuity of a function, and apply the Intermediate Value Theorem to continuous functions.
- Use limits to describe end behavior of functions.

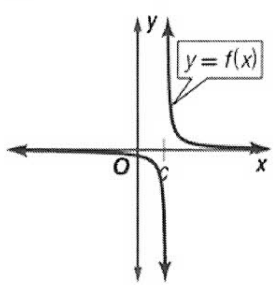
What does the term continuous mean?

The graph of a **continuous function** has no breaks, hole, or gaps.

KeyConcept Types of Discontinuity

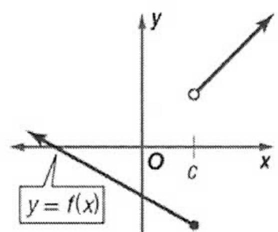
A function has an **infinite discontinuity** at $x = c$ if the function value increases or decreases indefinitely as x approaches c from the left and right.

Example



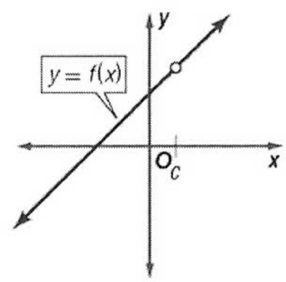
A function has a **jump discontinuity** at $x = c$ if the limits of the function as x approaches c from the left and right exist but have two distinct values.

Example



A function has a **removable discontinuity** if the function is continuous everywhere except for a hole at $x = c$.

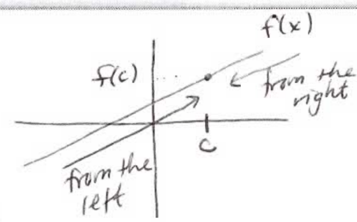
Example



ConceptSummary Continuity Test

A function $f(x)$ is continuous at $x = c$ if it satisfies the following conditions.

- $f(x)$ is defined at c. That is, $f(c)$ exists.
- $f(x)$ approaches the same value from either side of c. That is, $\lim_{x \to c} f(x)$ exists.
- The value that $f(x)$ approaches from each side of c is $f(c)$. That is, $\lim_{x \to c} f(x) = f(c)$.



EXAMPLE 1 Identify a Point of Continuity

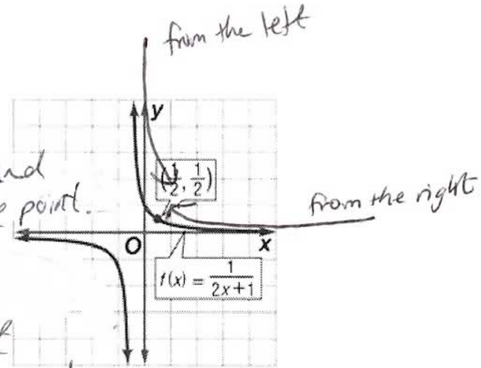
Determine whether $f(x) = \dfrac{1}{2x+1}$ is continuous at $x = \dfrac{1}{2}$. Justify using the continuity test.

Check the three conditions in the continuity test.

1. Does $f\left(\dfrac{1}{2}\right)$ exist?

$f(\frac{1}{2}) = \dfrac{1}{2(\frac{1}{2})+1} = \dfrac{1}{1+1} = \dfrac{1}{2}$ $\quad f(\frac{1}{2})$ exists.

2. Does $\lim_{x \to \frac{1}{2}} f(x)$ exist?



Yes, coming from the left and right you approach the same point.

3. Does $\lim_{x \to \frac{1}{2}} f(x) = \frac{1}{2}$?

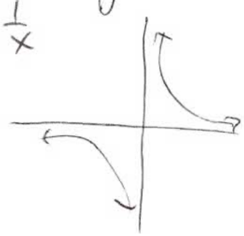
yes, coming from the left and the right you approach $y = \frac{1}{2}$

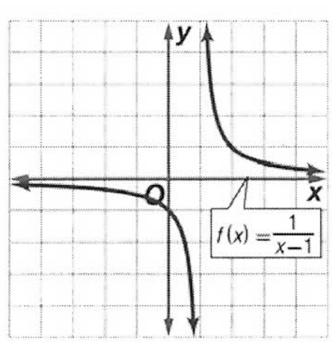
EXAMPLE 2 Identify a Point of Discontinuity

A. Determine whether the function $f(x) = \frac{1}{x-1}$ is continuous at $x = 1$. Justify using the continuity test. If discontinuous, identify the type of discontinuity as *infinite*, *jump*, or *removable*.

$f(1) = \frac{1}{1-1} = \frac{1}{0}$ f(1) does not exist

This is an infinite discontinuity because of the parent function $\frac{1}{x}$



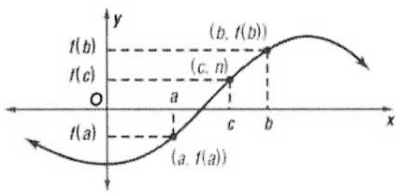


Can you draw a **continuous function** through the given points that is....

A. Even with two zeros B. Odd with three zeros C. Even with 4 zeros

KeyConcept Intermediate Value Theorem

If $f(x)$ is a continuous function and $a < b$ and there is a value n such that n is between $f(a)$ and $f(b)$, then there is a number c, such that $a < c < b$ and $f(c) = n$.



Corollary: The Location Principle If $f(x)$ is a continuous function and $f(a)$ and $f(b)$ have opposite signs, then there exists at least one value c, such that $a < c < b$ and $f(c) = 0$. That is, there is a zero between a and b.

Intermediate Value Theorm - Khan Academy

EXAMPLE 3 Approximate Zeros

A. Determine between which consecutive integers the real zeros of $f(x) = x^2 - x - \frac{3}{4}$ are located on the interval [–2, 2].

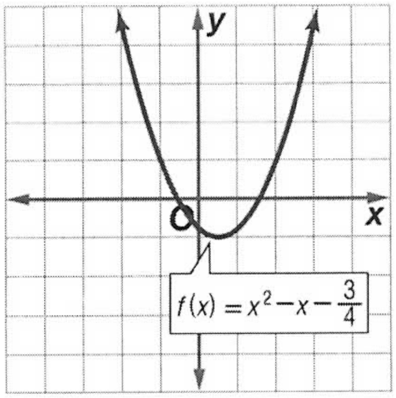
Investigate function values on the interval [–2, 2].

x	–2	–1	0	1	2
$f(x)$	5.25	1.25	–0.75	–0.75	1.25

↑ here ↑ here

* only way to go from negative → positive or positive → negative is to go through 0.

Verify with the graph.



You Try!

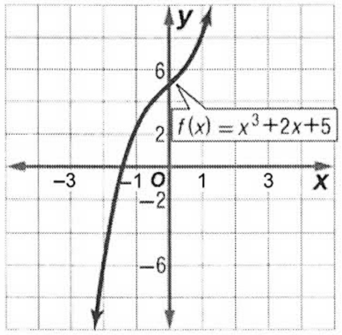
EXAMPLE 3 Approximate Zeros

B. Determine between which consecutive integers the real zeros of $f(x) = x^3 + 2x + 5$ are located on the interval [–2, 2].

Investigate function values on the interval [–2, 2].

x	–2	–1	0	1	2
$f(x)$	–7	2	5	8	17

here

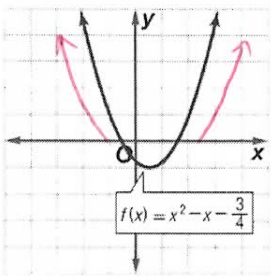


The end behavior of a function describes how a function behaves at either end of the graph as x approaches $\pm\infty$.

Describe the end behavior of the graph above.

$\lim\limits_{x \to -\infty} f(x) = -\infty$ $\lim\limits_{x \to +\infty} f(x) = +\infty$

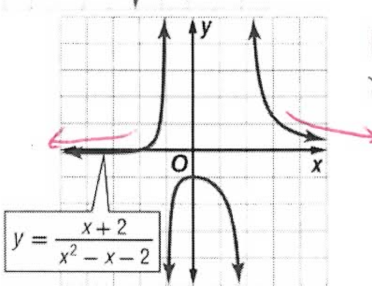
Describe the end behavior of each graph using Limit notation.



$\lim\limits_{x \to -\infty} f(x) = +\infty$

$\lim\limits_{x \to +\infty} f(x) = +\infty$

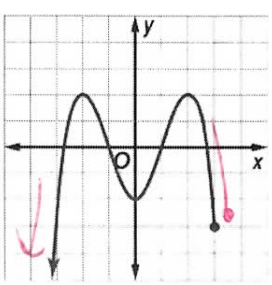
so $\lim\limits_{x \to \pm\infty} f(x) = \infty$



$\lim\limits_{x \to -\infty} f(x) = 0$

$\lim\limits_{x \to +\infty} f(x) = 0$

so $\lim\limits_{x \to \pm\infty} f(x) = 0$



$\lim\limits_{x \to -\infty} f(x) = -\infty$

$\lim\limits_{x \to 3} f(x) = -3$

Challenge!

Sketch a possible graph of $f(x)$ with the following characteristics.

$f(x)$ is a function and $f(2) = 4$

Domain: $(-\infty, 2] \cup (2, \infty)$ Range: $(-\infty, -4] \cup (1, \infty)$

$\lim_{x \to 2^-} f(x) = -4$ $\lim_{x \to 2^+} f(x) = 1$

$\lim_{x \to \infty} f(x) = \infty$ $\lim_{x \to -\infty} f(x) = -\infty$

Is $f(x)$ continuous? Explain how you know.

$f(2) = 4$ $f(2)$ is defined

$\lim_{x \to 2^-} f(x) = -4$

$\lim_{x \to 2^+} f(x) = 1$

$\lim_{x \to 2} f(x)$ does not exist.

The function is not continuous