Precalculus Final Exam Review

Name: Key

1.) Simplify: $\dfrac{1}{\csc\theta} + \dfrac{1}{\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin\theta + \dfrac{1}{\sin\theta}$

$\dfrac{\sin^2\theta}{\sin\theta} + \dfrac{1}{\sin\theta} = \dfrac{\sin^2\theta + 1}{\sin\theta}$

2.) Simplify: $\dfrac{\cos^2 x}{1-\csc^2 x}$

$1+\cot^2 x = \csc^2 x$

$1-\csc^2 x = -\cot^2 x$

$\dfrac{\cos^2 x}{-\cot^2 x} = \dfrac{\cos^2 x}{-\frac{\cos^2 x}{\sin^2 x}} = \cos^2 x \cdot \dfrac{-\sin^2 x}{\cos^2 x}$

$= -\sin^2 x$

3.) Find all solutions in the interval $[0,2\pi)$: $\csc x + 2 = 0$

$\csc x = -2$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

4.) Find all solutions in the interval $[0,2\pi)$: $2\cos^3 x + \cos^2 x = 0$

$\cos^2 x(2\cos x + 1) = 0$

$\cos^2 x = 0$ $\quad$ $2\cos x + 1 = 0$

$\cos x = 0$ $\quad$ $\cos x = -\frac{1}{2}$

$x = \pi/2, 3\pi/2$ $\quad$ $x = 2\pi/3, 4\pi/3$

5.) Evaluate: cos 165° (Hint: 165° = 210° - 45°)

$\cos(165) = \cos 210 \cos 45 + \sin 210 \sin 45$

$= -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + -\frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

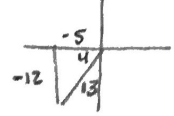
$-\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \dfrac{-\sqrt{6}-\sqrt{2}}{4}$

6.) Simplify: cos 6x cos 3x – sin 6x sin 3x

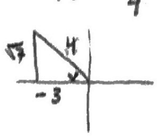
$\cos(6x+3x)$

$\cos(9x)$

7.) Given $\sin u = \dfrac{-12}{13}$, $\pi < u < \dfrac{3\pi}{2}$ and $\csc v = \dfrac{4}{\sqrt{7}}$, $\dfrac{\pi}{2} < v < \pi$, find cos (u + v)



$\sin v = \frac{\sqrt{7}}{4}$

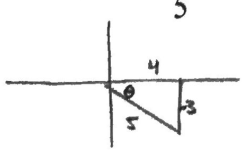


$\cos(u+v) = \cos u \cos v - \sin u \sin v$

$= -\frac{5}{13} \cdot -\frac{3}{4} - \frac{-12}{13} \cdot \frac{\sqrt{7}}{4}$

$= \frac{+15}{52} + \frac{12\sqrt{7}}{52} = \boxed{\dfrac{15+12\sqrt{7}}{52}}$

8.) Given $\cos\theta = \dfrac{4}{5}$ and $\sin\theta < 0$, find $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$



$= \dfrac{2(-\frac{3}{4})}{1-(-\frac{3}{4})^2} = \dfrac{-\frac{6}{4}}{1-\frac{9}{16}} = \dfrac{-\frac{6}{4}}{\frac{7}{16}} = -\frac{6}{4} \cdot \frac{16}{7} = \boxed{-\frac{24}{7}}$

9.) Find an equation of a line that passes through (5, 1) and is perpendicular to the line 4x – 2y = 5

$-2y = 5 - 4x$

$y = \dfrac{5-4x}{-2}$

$y = 2x - 5/2$

perpendicular $m = -\frac{1}{2}$

$y = -\frac{1}{2}x + b$

$1 = -\frac{1}{2}(5) + b$

$1 = -\frac{5}{2} + b$

$\frac{7}{2} = b$

$\boxed{y = -\frac{1}{2}x + \frac{7}{2}}$

10.) Given $f(x) = \begin{cases} x+3, x < -3 \\ |2x-8|, x \geq -3 \end{cases}$ find f(-5)

$-5+3 = \boxed{-2}$

11.) Find the vertex of the parabola: $y = x^2 - 2x + 8$

$x = -\frac{b}{2a}$

$x = -\frac{-2}{2(1)} = 1$

$y = (1)^2 - 2(1) + 8 = 7$

$(1,7)$

12.) Find the x- and y-intercepts of: $y = 2x^2 - 5x - 3$

$y = (2x+1)(x-3)$

$x = -\frac{1}{2}$ $\quad x = 3$

$\boxed{\text{x-int: } (-\tfrac{1}{2},0),(3,0) \quad \text{y-int: } (0,-3)}$

13.) Find the vertical asymptote(s): $f(x) = \dfrac{1}{(x-2)(2x+3)}$

$\boxed{x = 2 \quad x = -\frac{3}{2}}$

14.) Find the horizontal asymptote(s): $f(x) = \dfrac{2x^2 - 9}{3x^2 - 1}$

$\boxed{x = \frac{2}{3}}$

15.) The domain of $f(x) = 5 + e^x$

$\boxed{x = \mathbb{R}}$

16.) Convert from rectangular to polar coordinates: $\underbrace{x^2 + y^2}_{r^2} + 3\underbrace{x}_{r\cos\theta} - 2\underbrace{y}_{r\sin\theta} = 0$

$r^2 + 3r\cos\theta - 2r\sin\theta = 0$

$r(r + 3\cos\theta - 2\sin\theta) = 0$

$r + 3\cos\theta - 2\sin\theta = 0$

$\boxed{r = 2\sin\theta - 3\cos\theta}$

17.) Evaluate: $3\log_b \frac{1}{b}$

$\log_b \left(\frac{1}{b}\right)^3 = y$

$b^y = \frac{1}{b^3}$

$\boxed{y = -3}$

18.) Solve for x: $27^x = 243$

$\log_{27} 243 = x$

$\boxed{\frac{5}{3} = x}$

19.) Find a formula for the nth term of the sequence. (Assume n begins with 1)

$\frac{2}{1}, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \ldots$

$\boxed{a_n = \frac{n+1}{n^2}}$

20.) Find a_n for the arithmetic sequence with $a_1 = 3$, $d = -7$, and $n = 54$

$a_{54} = 3 + (54-1)\cdot -7 = \boxed{-368}$

21.) Find the sum of the infinite geometric sequence: 2, 1, 0.5, 0.25, …

$S = \frac{2}{1-\frac{1}{2}} = \frac{2}{\frac{1}{2}} = \boxed{4}$

22.) Eliminate the parameter and find the corresponding rectangular coordinates.

$x = 4\cos\theta$, $y = 3\sin\theta$

$\frac{x}{4} = \cos\theta$ $\quad \frac{y}{4} = \sin\theta$

$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

$\frac{x^2}{16} + \frac{y^2}{9} = 1$

23.) Write the point $\left(3, \frac{5\pi}{3}\right)$ in polar coordinates using three different representations.

$\left(3, -\frac{\pi}{3}\right)$ $\quad \left(-3, -\frac{4\pi}{3}\right)$

$\left(-3, \frac{2\pi}{3}\right)$

24.) Convert from polar to rectangular coordinates: $\left(\overset{r}{-2}, \overset{\theta}{\frac{7\pi}{6}}\right)$

$x = -2\cos\frac{7\pi}{6}$ $\quad y = -2\sin\frac{7\pi}{6}$

$= -2\left(-\frac{\sqrt{3}}{2}\right)$ $\quad = -2\left(-\frac{1}{2}\right)$

$= \sqrt{3}$ $\quad = 1$

$(\sqrt{3}, 1)$

25.) Convert from polar to rectangular coordinates: $r\cos^2\theta = 2\sin\theta$

$r = \sqrt{x^2+y^2}$

$x = r\cos\theta$

$y = r\sin\theta$

$r^2\cos^2\theta = 2r\sin\theta$

$x^2 = 2y$

$\boxed{y = \frac{x^2}{2}}$

26.) If $f(x) = \begin{cases} 3, x \le 2 \\ 5, x > 2 \end{cases}$ find $\lim_{x\to 2} f(x)$

DNE

27.) Find $\lim_{x\to\infty}\left(\frac{-3x^2+5}{2x^2-3x+1}\right) = -\frac{3}{2}$

28.) Find $\lim_{x\to 2}\left(\frac{2x^2-3x-2}{x-2}\right) = \frac{(2x+1)\cancel{(x-2)}}{\cancel{x-2}}$

$= 2(2)+1$

5

29.) Find $\lim_{x\to 4}\sqrt{x^2+2} = \sqrt{(4)^2+2}$

$= \sqrt{18}$

$= 3\sqrt{2}$

30.) Find an angle coterminal to $\theta = \frac{-4\pi}{3}$

$\frac{2\pi}{3}$

31.) Find the angle supplementary to $\theta = \frac{3\pi}{7}$

$2\pi - \frac{3\pi}{7}$

$\frac{14\pi}{7} - \frac{3\pi}{7} = \frac{11\pi}{7}$

32.) Convert to degrees: $\frac{6\pi}{5}$

$\frac{6\pi}{5} \cdot \frac{180}{\pi} = 216°$

33.) Convert to radians: 40°

$40 \cdot \frac{\pi}{180} = \frac{2\pi}{9}$

34.) Give the exact value: $\csc\left(\frac{-7\pi}{6}\right)$

$= \frac{1}{\sin(-\frac{7\pi}{6})}$

$= \frac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{-2\sqrt{3}}{3}$

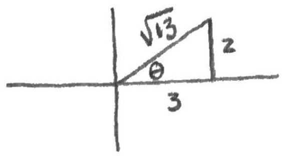
35.) Find θ if $\sec\theta = \frac{2\sqrt{3}}{3}$

$\cos\theta = \frac{3}{2\sqrt{3}}$

$\theta = \cos^{-1}\left(\frac{3}{2\sqrt{3}}\right)$

$\theta = 30°$

36.) A right triangle has an acute angle θ, such that $\tan\theta = \frac{2}{3}$. Find $\sin\theta$



$\sin\theta = \frac{2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}}$

$= \frac{2\sqrt{13}}{13}$

37.) Find the quadrant in which θ lies if $\tan\theta < 0$ and $\cos\theta > 0$

sin & cos opposite signs

1st or 4th quad.

4th

38.) Determine the period of f(x) if $f(x) = 2\cos\left(3x - \frac{\pi}{2}\right)$

$per = \frac{2\pi}{|3|}$

$= \frac{2\pi}{3}$

39.) Determine the amplitude of f(x) if $f(x) = -2\sin(4x + \pi)$

amp= 2

40.) Describe the horizontal shift to the graph of g(x), given $g(x) = 3\sin\left(2x - \frac{\pi}{4}\right)$

right $\frac{\pi}{4}$.

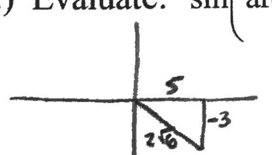
41.) Determine the period of the function: f(x) = 4 tan(5x)

$per = \frac{\pi}{|5|}$

$per = \frac{\pi}{5}$

domain=$(-\pi, \pi)$

42.) Evaluate: $\sin\left(\arctan\left(\frac{-3}{5}\right)\right)$



$= -\frac{3}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{-3\sqrt{6}}{12} = -\frac{\sqrt{6}}{4}$

43.) Simplify: $\frac{1-\sin x}{1-\sin x} \cdot \frac{1}{1+\sin x} + \frac{1}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x}$

$\frac{1-\sin x + 1 + \sin x}{1 - \sin^2 x} = \frac{2}{\cos^2 x} = 2\sec^2 x$

44.) Solve for x: $\log(5 - x) - \log(2x - 6) = 1$

$\log \frac{5-x}{2x-6} = 1$

$10^1 = \frac{5-x}{2x-6}$

$10 = \frac{5-x}{2x-6}$

$5 - x = 20x - 60$

$65 = 21x$

$\boxed{3.095 = x}$

45.) Find the vertices of the hyperbola: $\frac{(x-3)^2}{4} - \frac{(y+1)^2}{16} = 1$

center: $(3, -1)$

$a = 2$

vertices: $(5, -1), (1, -1)$

46.) Find the center of the ellipse: $4x^2 + 5y^2 + 16x - 10y + 1 = 0$

$4x^2 + 16x + 5y^2 - 10y = -1$

$4(x^2 + 4x) + 5(y^2 - 2y) = -1$

$4(x+2)^2 - 4(4) + 5(y-1)^2 - 5(1) = -1$

$\frac{4(x+2)^2}{20} + \frac{5(y-1)^2}{20} = \frac{20}{20}$

$\frac{(x+2)^2}{5} + \frac{(y-1)^2}{4} = 1$

Center: $(-2, 1)$

47.) A vector w has initial point (4, 6) and terminal point (2, -5). Find the x- and y- components of the vector.

$\langle 2-4, -5-6 \rangle$

$\langle -2, -11 \rangle$

48.) Solve $|x - 5| \le 10$

$x - 5 \le 10$

$x \le 15$

$-(x-5) \le 10$

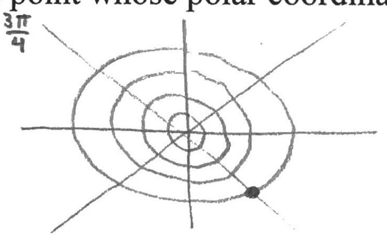
$-x + 5 \le 10$

$-x \le 5$

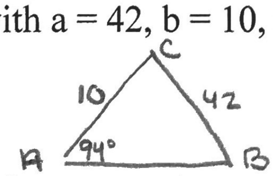
$x \ge 5$

49.) Plot the point whose polar coordinates are $\left(-4, \frac{3\pi}{4}\right)$

$\frac{3\pi}{4}$



50.) Given a triangle with a = 42, b = 10, and A = 94°, find angle C.



$\frac{\sin 94}{42} = \frac{\sin B}{10}$

$B = 13.74°$

$C = 180 - 94 - 13.74$

$= 72.26°$

51.) Find the number of years required for a \$3500 investment to triple at a 7% interest rate compounded continuously.

$\frac{10{,}500}{3500} = \frac{3{,}500e^{0.07t}}{3500}$

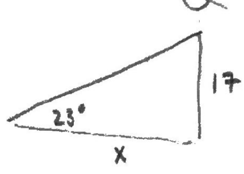
$3 = e^{0.07t}$

$\ln 3 = 0.07t$

15.7yrs $= t$

52.) The sun is 23° above the horizon. Find the length of a shadow cast by a flagpole 17 feet tall.

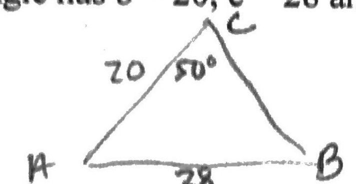
α



$\tan(23°) = \frac{17}{x}$

$x = 40$ ft

53.) A triangle has b = 20, c = 28 and C = 50°. Find the area of the triangle.



$\frac{\sin 50}{28} = \frac{\sin B}{20}$

$B = 33.17°$

$A = 96.83°$

area $= \frac{1}{2} bc \sin A$

$= \frac{1}{2}(20)(28)\sin(96.83)$

$= 278$

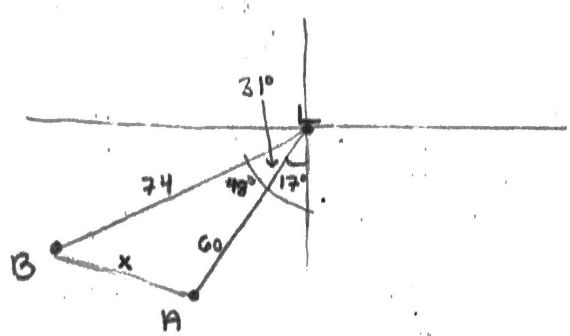
54.) A triangle has a = 50.2 cm, b = 29.7 cm, and c = 63 cm. Find the area. (Heron's Formula)

$s = \frac{1}{2}(50.2 + 29.7 + 63) = 71.45$

area $= \sqrt{71.45(71.45-50.2)(71.45-29.7)(71.45-63)}$

$= 733.42$

55.) Ship A is 60 miles from a lighthouse on shore. Its bearing from the lighthouse is S 17° W. Ship B is 74 miles from the same lighthouse with a bearing of S 48° W. Find the number of miles between the ships.



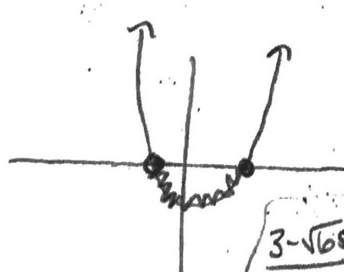
$x^2 = 60^2 + 74^2 - 2(60)(74)\cos(31°)$

$x = 38.27$ mi

56.) Solve $2x^2 - 3x - 7 < 0$

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-7)}}{2(2)}$

$x = \frac{3 \pm \sqrt{65}}{4}$



$\frac{3-\sqrt{65}}{4} < x < \frac{3+\sqrt{65}}{4}$

57.) Find all ***exact, real solutions*** of $4x^3 - 38x - 6 = 0$.

$2x^3 - 19x - 3 = 0$

-3	2	0	-19	-3
	↓	-6	18	3
	2	-6	-1	0

$(x-3)(2x^2 - 6x - 1) = 0$

$x = 3$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(-1)}}{2(2)}$

$x = \frac{-6 \pm \sqrt{44}}{4}$

$x = \frac{-6 \pm 2\sqrt{11}}{4}$

$x = \frac{-3 \pm \sqrt{11}}{2}$