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\# 33-47, 51-55, 63 odds only

33. $\frac{(1+\cos x)}{(1+\cos x)} \cdot \frac{1-\cos x}{\tan x} + \frac{\sin x}{1+\cos x} \cdot \frac{(\tan x)}{(\tan x)}$

$$\frac{(1+\cos x)(1-\cos x) + \sin x \cdot \tan x}{(1+\cos x)\tan x}$$

$$\frac{(1-\cos^2 x) + \sin x \cdot \tan x}{(1+\cos x)\tan x}$$

$$\frac{\sin^2 x + \sin x \cdot \tan x}{(1+\cos x)\cdot \tan x}$$

$$\frac{\sin x(\sin x + \tan x)}{(1+\cos x)\tan x}$$

$$\frac{\cancel{\sin x}(\sin x + \tan x)}{(1+\cos x)\cdot \frac{\cancel{\sin x}}{\cos x}}$$

$$\frac{\sin x + \tan x}{(1+\cos x)\cdot \frac{1}{\cos x}}$$

$$\frac{\sin x + \frac{\sin x}{\cos x}}{(1+\cos x)\frac{1}{\cos x}}$$

$$\frac{\sin x\cancel{\left(1 + \frac{1}{\cos x}\right)}}{\cancel{\frac{1}{\cos x} + 1}} = \sin x$$

35. $\frac{(\sec x - \tan x)}{(\sec x - \tan x)} \cdot \frac{\cos x \cot x}{\sec x + \tan x} + \frac{\sin x}{\sec x - \tan x} \cdot \frac{(\sec x + \tan x)}{(\sec x + \tan x)}$

$$\frac{\sec x \cos x \cot x - \tan x \cancel{\cos x \cot x} + \sin x \sec x + \sin x \tan x}{\sec^2 x - \tan^2 x}$$

$$\frac{\frac{1}{\cancel{\cos x}} \cdot \cancel{\cos x} \cdot \frac{\cos x}{\sin x} - \frac{\cancel{\sin x}}{\cancel{\cos x}} \cdot \cancel{\cos x} \cdot \frac{\cos x}{\cancel{\sin x}} + \sin x \cdot \frac{1}{\cos x} + \sin x \cdot \frac{\sin x}{\cos x}}{\sec^2 x - \tan^2 x}$$

$$\frac{\frac{\cos x}{\sin x} - \cos x + \frac{\sin x}{\cos x} + \frac{\sin^2 x}{\cos x}}{\cancel{(\tan^2 x + 1) - \tan^2 x}}$$

$$\frac{\cos x}{\cos x} \cdot \frac{\cos x}{\sin x} - \cos x \frac{\sin x \cos x}{\sin x \cos x} + \frac{\sin x}{\cos x} \cdot \frac{\sin x}{\sin x} + \frac{\sin^2 x}{\cos x} \cdot \frac{\sin x}{\sin x}$$

$$\frac{\cos^2 x}{\cos x \sin x} - \frac{\sin x \cos^2 x}{\sin x \cos x} + \frac{\sin^2 x}{\sin x \cos x} + \frac{\sin^3 x}{\cos x \sin x}$$

$$\frac{\cos^2 x - \sin x \cos^2 x + \sin^2 x + \sin^3 x}{\sin x \cos x}$$

$$\frac{1 - \sin x \cos^2 x + \sin^3 x}{\sin x \cos x}$$

$$\frac{1 - \sin x(\cos^2 x + \sin^2 x)}{\sin x \cos x}$$

$$\frac{1 - \sin x}{\sin x \cos x}$$

37. a. $I = I_0 - \dfrac{I_0}{\csc^2\theta}$

$I = \dfrac{\csc^2\theta\, I_0}{\csc^2\theta} - \dfrac{I_0}{\csc^2\theta}$

$\cot^2\theta + 1 = \csc^2\theta$

$\cot^2\theta = \csc^2\theta - 1$

$I = \dfrac{\csc^2\theta\, I_0 - I_0}{\csc^2\theta}$

$I = \dfrac{I_0(\csc^2\theta - 1)}{\csc^2\theta}$

$I = \dfrac{I_0 \cot^2\theta}{\csc^2\theta} = I_0 \dfrac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{1}{\sin^2\theta}} = I_0 \dfrac{\cos^2\theta}{\cancel{\sin^2\theta}} \cdot \dfrac{\cancel{\sin^2\theta}}{1} = I_0 \cos^2\theta$

b. $I = I_0 \cos^2(30^\circ)$

$= I_0 (\cos(30^\circ))^2$

$= I_0 \left(\frac{\sqrt{3}}{2}\right)^2$

$= I_0 \frac{3}{4}$

38. $\dfrac{\sin x}{\csc x - \cot x} = \dfrac{\sin x}{\frac{1}{\sin x} - \frac{\cos x}{\sin x}} = \dfrac{\sin x}{\frac{1-\cos x}{\sin x}} = \sin x \cdot \dfrac{\sin x}{1-\cos x} = \dfrac{\sin^2 x}{1-\cos x} = \dfrac{1-\cos^2 x}{1-\cos x} = \dfrac{(1+\cos x)(1+\cos x)}{\cancel{1-\cos x}}$

$= 1 + \cos x$

39. $\dfrac{\csc x}{1-\sin x} = \dfrac{\frac{1}{\sin x}}{1-\sin x} = \dfrac{1}{\sin x} \cdot \dfrac{1}{1-\sin x} = \dfrac{1}{\sin x - \sin^2 x}$

$= \dfrac{1}{\sin x - (1-\cos^2 x)} = \dfrac{1}{\sin x - 1 + \cos^2 x}$

39. $\dfrac{\csc x}{1-\sin x} \cdot \dfrac{1+\sin x}{1+\sin x} = \dfrac{\csc x + \csc x \sin x}{1-\sin^2 x} = \dfrac{\csc x + \csc x \sin x}{\cos^2 x} = \dfrac{\csc x + 1}{\cos^2 x} = \dfrac{1}{\cos^2 x} \cdot (\csc x + 1)$

$= \sec^2 x(\csc x + 1)$

41. $\dfrac{\cot x}{1+\sin x} \cdot \dfrac{1-\sin x}{1-\sin x} = \dfrac{\cot x - \cot x \sin x}{1-\sin^2 x} = \dfrac{\cot x - \cot x \sin x}{\cos^2 x}$

$= \dfrac{\cot x}{\cos^2 x} - \dfrac{\cot x \sin x}{\cos^2 x}$

$= \dfrac{\frac{\cos x}{\sin x}}{\cos^2 x} - \dfrac{\frac{\cos x}{\sin x} \cdot \sin x}{\cos^2 x}$

$= \dfrac{\cos x}{\sin x} \cdot \dfrac{1}{\cos^2 x} - \dfrac{\cos x}{\cos^2 x}$

$= \dfrac{1}{\sin x \cos x} - \dfrac{1}{\cos x}$

$= \csc x \sec x - \sec x = \sec x(\csc x - 1)$

43. $\dfrac{2\sin x}{\cot x + \csc x} \cdot \dfrac{\cot x - \csc x}{\cot x - \csc x} = \dfrac{2\sin x \cot x - 2\sin x \csc x}{\cot^2 x - \csc^2 x}$

$= \dfrac{2\sin x \cdot \frac{\cos x}{\sin x} - 2\sin x \cdot \frac{1}{\sin x}}{\cot^2 x - (\cot^2\theta + 1)}$

$= \dfrac{2\cos x - 2}{-1} = \dfrac{2(\cos x - 1)}{-1} = -2(\cos x - 1)$

45. $\dfrac{\cot^2 x \cos x}{\csc x - 1} \cdot \dfrac{\csc x + 1}{\csc x + 1} = \dfrac{\cot^2 x \cos x \csc x + \cot^2 x \cos x}{\csc^2 x - 1}$

$= \dfrac{\frac{\cos^2 x}{\sin^2 x} \cos x \cdot \frac{1}{\sin x} + \frac{\cos^2 x}{\sin^2 x} \cdot \cos x}{\cot^2 x}$

$= \dfrac{\frac{\cos^3 x}{\sin^3 x} + \frac{\cos^3 x}{\sin^2 x}\left(\frac{\sin x}{\sin x}\right)}{\cot^2 x} = \dfrac{\frac{\cos^3 x + \cos^3 x \sin x}{\sin^3 x}}{\frac{\cos^2 x}{\sin^2 x}}$

$= \dfrac{\cos^3 x + \cos^3 x \sin x}{\sin^3 x} \cdot \dfrac{\sin^2 x}{\cos^2 x}$

$= \dfrac{\cancel{\cos^2 x}(\cos x + \cos x \sin x)}{\sin^3 x} \cdot \dfrac{\sin^2 x}{\cancel{\cos^2 x}}$

$\dfrac{\cos x + \cos x \sin x}{\sin x}$

$\cot x + \cos x$

47. $\frac{\sin x \tan x}{\cos x + 1} \cdot \frac{\cos x - 1}{\cos x - 1} = \frac{\sin x \cos x \tan x - \sin x \tan x}{\cos^2 x - 1} = \frac{\sin x \cos x \frac{\sin x}{\cos x} - \sin x \frac{\sin x}{\cos x}}{-\sin^2 x}$

$= \frac{\sin^2 x - \frac{\sin^2 x}{\cos x}}{-\sin^2 x}$

$= \frac{\frac{\sin^2 x \cos x}{\cos x} - \frac{\sin^2 x}{\cos x}}{-\sin^2 x} = \frac{\frac{\cancel{\sin^2 x}(\cos x - 1)}{\cos x}}{-\cancel{\sin^2 x}} = \frac{\cos x - 1}{\cos x} = 1 - \sec x$

51. $\tan x - \csc x \sec x = \frac{\sin x}{\cos x} - \frac{1}{\sin x} \cdot \frac{1}{\cos x}$

$\frac{\sin^2 x}{\sin x \cos x} - \frac{1}{\sin x \cos x} = \frac{\sin^2 x - 1}{\sin x \cos x} = \frac{-\cos^2 x}{\sin x \cos x} = \frac{-\cos x}{\sin x}$

$= -\cot x$

53. $\csc x \tan^2 x - \sec^2 x \csc x = \frac{1}{\sin x} \cdot \frac{\sin^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} \cdot \frac{1}{\sin x} = \frac{\sin^2 x - 1}{\cos^2 x \sin x} = \frac{-\cancel{\cos^2} x}{\cancel{\cos^2 x} \sin x}$

$= -\csc x$

55. a.

X	-2π	$-\pi$	0	π	2π
Y_1	1	1	1	1	1
Y_2	1	1	1	1	1
Y_3	0	0	0	0	0
Y_4	0	0	0	0	0

$Y_1 = \tan x + 1$

$Y_2 = \frac{1}{\cos x} \cdot \cos x - \sin x \cdot \frac{1}{\cos x} = 1 - \tan x$

$Y_3 = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} - \cancel{\sin x}$

$\frac{\sin x}{\cos^2 x} - \frac{\cos^2 x \sin x}{\cos^2 x} = \frac{\sin x(1-\cos^2 x)}{\cos^2 x} = \frac{\sin^3 x}{\cos^2 x}$

$= \sin x \tan^2 x$

$Y_4 = \sin x \tan^2 x$

c. $Y_1 = Y_2$, $Y_3 = Y_4$

d. no, this is not true

$Y_1 \neq Y_2$

but $Y_3 = Y_4$

63. $\frac{1-\sin^2 x}{\sin^2 x-\cos^2 x} = \frac{\cos^2 x}{\sin^2 x-\cos^2 x} \rightarrow \frac{\cos^2 x}{(1-\cos^2 x)-\cos^2 x} = \frac{\cos^2 x}{1-2\cos^2 x}$ ✓

$\frac{\cos^2 x}{\sin^2 x-(1-\sin^2 x)} = \frac{\cos^2 x}{2\sin^2 x-1}$

Janelle is right.