5.3 Solving Trig Equations Practice Worksheet #1
Pre-calculus

Name: _____
Date: _____ Block: ____

Solve for the unknown variable on the interval $0 \le x < 2\pi$.

1. $4\cos^2 x - 3 = 0$

$+3 \quad +3$

$\frac{4\cos^2 x}{4} = \frac{3}{4}$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm\sqrt{3/4} = \pm\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

2. $\sqrt{2}\sin 2x = 1$

$\sin 2x = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\sin 2x = \frac{\sqrt{2}}{2}$

$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = 2x$

$\frac{\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)}{2} = x$

$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}, \frac{3\pi}{4}$

$\frac{\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)}{2} = \frac{\pi}{8}, \frac{3\pi}{8}$

3. $3\cot^2 x - 1 = 0$

$3\cot^2 x = 1$

$\cot^2 x = \frac{1}{3}$

$\cot x = \frac{1}{\sqrt{3}}$

$\tan x = \frac{\sqrt{3}}{1}$

$x = \frac{\pi}{3}, \frac{4\pi}{3}$

$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} \begin{matrix}\sin \\ \cos\end{matrix} = \frac{\sqrt{3}}{1}$

4. $\frac{\cos^3 x}{\cos x} = \frac{\cos x}{\cos x}$

$\cos^2 x = 1$

$\cos x = \pm 1$

$x = 0, \pi, 2\pi$

5. $\sin x - 2\sin x\cos x = 0$

$-2\sin x\cos x = -\sin x$

$\frac{2\sin x\cos x}{\sin x} = \frac{\sin x}{\sin x}$

$2\cos x = 1$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

6. $2\sin^2 x - \sin x - 3 = 0$ $\quad x = \sin x$

$2x^2 - x - 3 = 0$

$(2x - 3)(x + 1) = 0$

$2x - 3 = 0 \qquad x + 1 = 0$

$2x = 3 \qquad -1 \ -1$

$x = \frac{3}{2} \qquad x = -1$

$\sin x = \frac{3}{2}$ ↑ no solution

$\sin x = -1$

$x = \frac{3\pi}{2}$

sin always < 1

7. $\csc^2 x - \csc x - 2 = 0$ $\quad x = \csc x$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2 \qquad x = -1$

$\csc x = 2 \qquad \csc x = -1$

$\frac{1}{\sin x} = 2 \qquad \frac{1}{\sin x} = -1$

$\sin x = \frac{1}{2} \qquad \sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6} \qquad x = \frac{3\pi}{2}$

8. $\cos^2 x = 1 - \sin x$

Skip.

Solve for the unknown variable on the given interval.

9. $\sqrt{3} + \tan(2x) = 0$ on $[0, 2\pi)$.

$\tan(2x) = -\sqrt{3}$

$\tan^{-1}(-\sqrt{3}) = 2x$

$\frac{\tan^{-1}(-\sqrt{3})}{2} = x$

$\tan^{-1}(-\sqrt{3}) = \tan^{-1}\left(\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)$ $\begin{matrix}\sin \\ \cos\end{matrix}$ or $\tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)$ $\begin{matrix}\sin \\ \cos\end{matrix}$

$\frac{5\pi}{3}$ $\qquad \frac{2\pi}{3}$

$\frac{\tan^{-1}(-\sqrt{3})}{2} = \frac{5\pi}{6}, \frac{2\pi}{6}$

10. $\cos(\pi x) = 0.5$ on $[0, 2)$.

$\cos^{-1}\left(\frac{1}{2}\right) = \pi x$

$\frac{\cos^{-1}\left(\frac{1}{2}\right)}{\pi} = x$

$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}, \frac{5\pi}{3}$

$\frac{\cos^{-1}\left(\frac{1}{2}\right)}{\pi} = \frac{1}{3}, \frac{5}{3}$

11. $\sin\left(\frac{x}{2}\right) - 1 = 0$ on $[0, 8\pi)$.

$\sin\left(\frac{x}{2}\right) = 1$

$\sin^{-1}(1) = \frac{x}{2}$

$2\sin^{-1}(1) = x$

$\sin^{-1}(1) = \frac{\pi}{2}, \frac{5\pi}{2}$

$2\sin^{-1}(1) = 2 \cdot \frac{\pi}{2} = \pi$

$2 \cdot \frac{5\pi}{2} = 5\pi$

5.3 Solving Trig Equations – Worksheet #2
Pre-calculus

Name: _____
Date: _____ Block: ___

Part 1: Solve for the unknown variable. Give all of the exact general solutions.

1. $\sin\theta = \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

2. $\cos\theta = \sin\theta$

$\theta = \pi/4, \frac{5\pi}{4}$

3. $\tan\theta = 1$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

4. $1 + \sin\theta = 2\cos^2\theta$

Skip

5. $2\cos^2\theta + \cos\theta = 0$

$\cos\theta(2\cos\theta + 1) = 0$

$\cos\theta = 0$ $\quad 2\cos\theta + 1 = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad 2\cos\theta = -1$

$\cos\theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

6. $\sin 3\theta = -1$

$\sin^{-1}(-1) = 3\theta$

$\frac{\sin^{-1}(-1)}{3} = \theta$

$\sin^{-1}(-1) = \frac{3\pi}{2}$

$\frac{\sin^{-1}(-1)}{3} = \frac{\frac{3\pi}{2}}{3} = \frac{3\pi}{6} = \frac{\pi}{2}$

7. $\sin^2\theta - 1 = 0$

$\sin^2\theta = 1$

$\sin\theta = \pm 1$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

8. $\cos 2\theta = \frac{1}{2}$

$\cos^{-1}(\frac{1}{2}) = 2\theta$

$\frac{\cos^{-1}(\frac{1}{2})}{2} = \theta$

$\cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}, \frac{5\pi}{3}$

$\frac{\cos^{-1}(\frac{1}{2})}{2} = \frac{\pi}{6}, \frac{5\pi}{6}$

9. $2\sin^2\theta - \sin\theta - 1 = 0$

$(2\sin\theta + 1)(\sin\theta - 1) = 0$

$2\sin\theta + 1 = 0$ $\quad \sin\theta - 1 = 0$

$\sin\theta = -\frac{1}{2}$ $\quad \sin\theta = 1$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$ $\quad \theta = \frac{\pi}{2}$

10. $\tan 4\theta = -1$

$\tan^{-1}(-1) = 4\theta$

$\frac{\tan^{-1}(-1)}{4} = \theta$

$\tan^{-1}(-1) = \frac{3\pi}{4}, \frac{7\pi}{4}$

$\frac{\tan^{-1}(-1)}{4} = \frac{3\pi}{16}, \frac{7\pi}{16}$

11. $\tan^2 3x = 3$

$\tan 3x = \pm\sqrt{3}$

$\tan^{-1}(\pm\sqrt{3}) = 3x$

$\frac{\tan^{-1}(\pm\sqrt{3})}{3} = x$

$\tan^{-1}(\pm\sqrt{3}) = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\frac{\tan^{-1}(\pm\sqrt{3})}{3} = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}$

12. $\cos\frac{x}{2} = \frac{\sqrt{2}}{2}$

$\cos^{-1}(\frac{\sqrt{2}}{2}) = \frac{x}{2}$

$2\cos^{-1}(\frac{\sqrt{2}}{2}) = x$

$\cos^{-1}(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}, \frac{7\pi}{4}$

$2\cos^{-1}(\frac{\sqrt{2}}{2}) = \frac{\pi}{2}, \frac{7\pi}{2}$