

#23-41 odd, 64-67, 81-84

23. possible real zeroes: 5

turning points: 4

$$f(x) = x^5 + 3x^4 + 2x^3$$

$$= x^3(x^2 + 3x + 2)$$

$$= x^3(x+1)(x+2)$$

$x = 0, -1, -2$

25. possible real zeroes: 4

turning points: 3

$$f(x) = x^4 + 4x^2 - 21$$

$$= (x^2 + 7)(x^2 - 3)$$

$$x = \pm i\sqrt{7}, \pm \sqrt{3}$$

27. possible real zeroes: 6

turning points: 5

$$f(x) = x^6 - 6x^3 - 16$$

$$= (x^3 - 8)(x^3 + 2)$$

$$x = \sqrt[3]{8}, \sqrt[3]{-2}$$

29. possible real zeroes: 6

turning points: 5

$$f(x) = 9x^6 - 36x^4$$

$$= 9x^4(x^2 - 4)$$

$$= 9x^4(x-2)(x+2)$$

$x = 0, 2, -2$

31. possible real zeroes: 4

turning points: 3

$$f(x) = 4x^4 - 4x^3 - 3x^2$$

$$= x^2(4x^2 - 4x - 3)$$

$$= x^2(2x+1)(2x-3)$$

$x = 0, -\frac{1}{2}, \frac{3}{2}$

	$2x$	$-3$	
$2x$	$4x^2$	$-6x$	$-12$
$1$	$2x$	$-3$	$-6+12$

33.  $f(x) = x(x+4)(x-1)^2$

a) leading term:  $x^4$

deg = 4, positive leading coefficient

$\lim_{x \rightarrow \pm\infty} f(x) = \infty$

b)  $x = 0, -4, 1$  (multiplicity = 2)

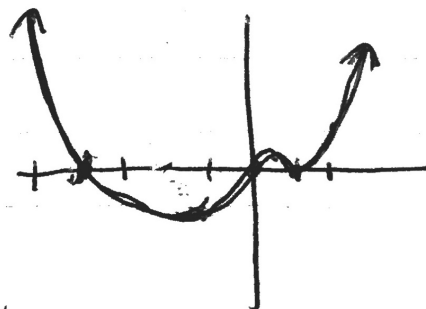
c) interval x-value  $f(x)$   $(x, f(x))$

$(-\infty, -4)$  -5 180  $(-5, 180)$

$(-4, 0)$  -1 -12  $(-1, -12)$

$(0, 1)$  0.5 0.5625  $(0.5, 0.5625)$

$(1, \infty)$  2 12  $(2, 12)$

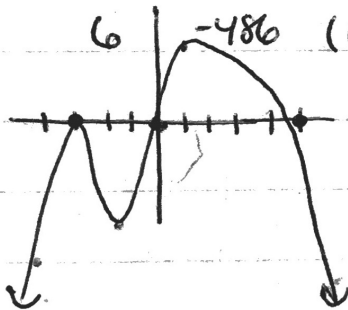


35.  $f(x) = -x(x+3)^2(x-5)$

a) leading term:  $-x^4$   
 even degree, negative coefficient  
 $\lim_{x \rightarrow \pm\infty} f(x) = -\infty$

b)  $x = 0, -3$  (multiplicity = 2),  $5$

interval	x-value	f(x)	(x, f(x))
$(-\infty, -3)$	-4	-36	$(-4, -36)$
$(-3, 0)$	-1	-24	$(-1, -24)$
$(0, 5)$	1	64	$(1, 64)$
$(5, \infty)$	6	-486	$(6, -486)$

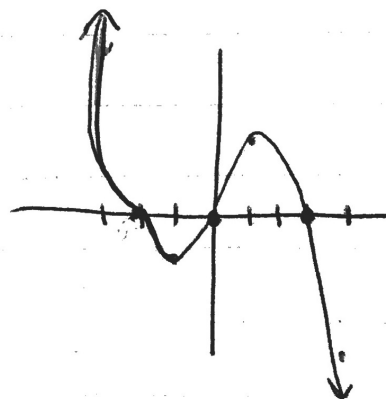


37.  $f(x) = -x(x-3)(x+2)^3$

a) leading term =  $-x^5$   
 odd degree, negative coefficient  
 $\lim_{x \rightarrow -\infty} f(x) = \infty, \lim_{x \rightarrow \infty} f(x) = -\infty$

b)  $x = 0, 3, -2$  (multiplicity = 3)

interval	x-value	f(x)	(x, f(x))
$(-\infty, -2)$	-3	+18	$(-3, +18)$
$(-2, 0)$	-1	-4	$(-1, -4)$
$(0, 3)$	1	18	$(1, 18)$
$(3, \infty)$	4	-144	$(4, -144)$



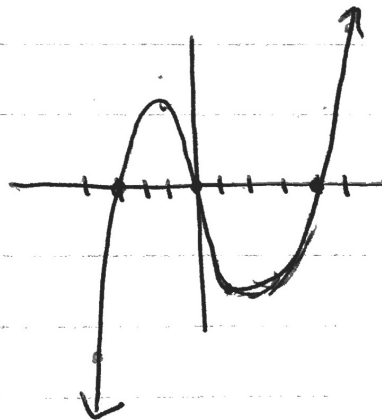
39.  $f(x) = 3x^3 - 3x^2 - 36x$

a) leading term =  $3x^3$ ; odd deg. + coeff  
 $\lim_{x \rightarrow -\infty} f(x) = -\infty, \lim_{x \rightarrow \infty} f(x) = \infty$

b)  $f(x) = 3x(x^2 - x - 12)$   
 $= 3x(x-4)(x+3)$

$x = 0, 4, -3$

interval	x-value	f(x)	(x, f(x))
$(-\infty, -3)$	-4	-96	$(-4, -96)$
$(-3, 0)$	-1	30	$(-1, 30)$
$(0, 4)$	1	-36	$(1, -36)$
$(4, \infty)$	5	120	$(5, 120)$



41.  $f(x) = x^4 + x^3 - 20x^2$

a) leading term:  $x^4$

even degree, + coeff.

$$\lim_{x \rightarrow \pm\infty} f(x) = \infty$$

b)  $f(x) = x^2(x^2 + x - 20)$   
 $= x^2(x+5)(x-4)$

$x = 0$  (multiplicity = 2),  $-5, 4$

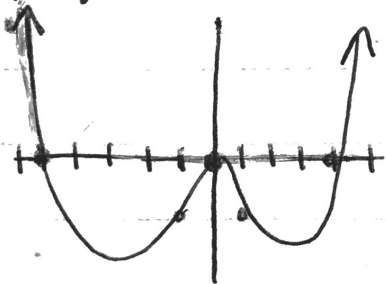
c) interval x-value f(x) (x, f(x))

$(-\infty, -5)$      $-6$      $360$      $(-6, 360)$

$(-5, 0)$      $-1$      $-20$      $(-1, -20)$

$(0, 4)$      $1$      $-18$      $(1, -18)$

$(4, \infty)$      $5$      $250$      $(5, 250)$



64. deg = even, leading coeff = ~~even~~ <sup>+</sup>

65. deg = odd, leading coeff = +

66. deg = odd, leading coeff = -

67. deg = even, leading coeff = -

81. a) deg = 4;  $\lim_{x \rightarrow \pm\infty} f(x) = \infty$

b)  $x = -6, -2$  (mult = 2),  $4$

c)  $y = a(x+6)(x+2)^2(x-4)$

$$-128 = a(2+6)(2+2)^2(2-4)$$

$$-128 = a(8)(4)^2(-2)$$

$$-128 = -256a$$

$$\frac{1}{2} = a$$

$$y = \frac{1}{2}(x+6)(x+2)^2(x-4)$$

82. a) deg = 4;  $\lim_{x \rightarrow \pm\infty} f(x) = -\infty$

b)  $x = -3, 2, 6$  (mult = 2)

c)  $y = a(x+3)(x-2)(x-6)^2$

$$64 = a(-2+3)(-2-2)(-2-6)^2$$

$$64 = -256a$$

$$-\frac{1}{4} = a$$

$$y = -\frac{1}{4}(x+3)(x-2)(x-6)^2$$

83. a) deg = 4;  $\lim_{x \rightarrow \pm\infty} f(x) = \infty$

b)  $x = -4, -1, 3$  (mult = 2)

c)  $y = a(x+4)(x+1)(x-3)^2$

$$-9 = a(-3+4)(-3+1)(-3-3)^2$$

$$-9 = -72a$$

$$\frac{1}{8} = a$$

$$y = \frac{1}{8}(x+4)(x+1)(x-3)^2$$

$$84. a) \text{deg} = 5; \lim_{x \rightarrow -\infty} f(x) = -\infty, \lim_{x \rightarrow \infty} f(x) = \infty$$

$$b) x = -3 (\text{mult} = 2), -1, 2 (\text{mult} = 2)$$

$$c) y = a(x+3)^2(x-1)(x-2)^2$$

$$3.6 = a(0+3)^2(0-1)(0-2)^2$$

$$3.6 = -36a$$

$$-\frac{1}{10} = a$$

$$y = -\frac{1}{10}(x+3)^2(x-1)(x-2)^2$$