Day 4

Wednesday, October 11, 2017
7:37 PM

10/13

4.2 Radians/Degrees

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PreCalculus Notes 4-2

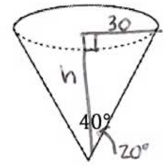
Copy Guided Notes

$V = \frac{1}{3}\pi(30)^2 \cdot 82.42$

$V = 77{,}679 \text{ cm}^3$

Warm-up:

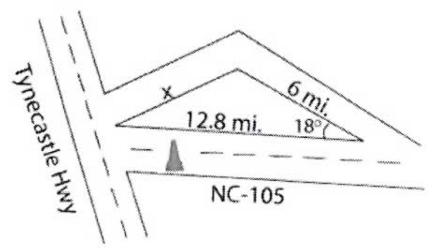
1. An open right-circular cone has a vertical angle measuring $40°$ and a base radius of 30 cm. Find the capacity of the cone in liters. ($V = \frac{1}{3}\pi r^2 h$)



$\tan 20 = \frac{30}{h}$

$h = 82.42$

2. You are heading to Beech Mountain for a ski trip. Unfortunately, state road 105 in North Carolina is blocked off due to a chemical spill. Assuming that both roads on the detour are straight, how many extra miles are you traveling to reach your destination?



Pasted from <http://www.algebralab.org/practice/practice.aspx?file=Word_LawofCosines.xml>

$x^2 = 12.8^2 + 6^2 - 2(12.8)(6)\cos(18)$

$x^2 = 53.7577$

$x = 7.33$

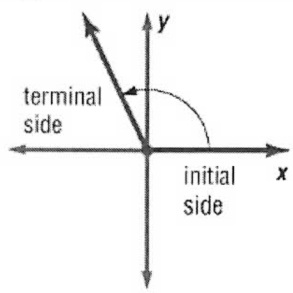
$7.33 + 6 = 13.33$

$13.33 - 12.8 = 0.53$ extra miles

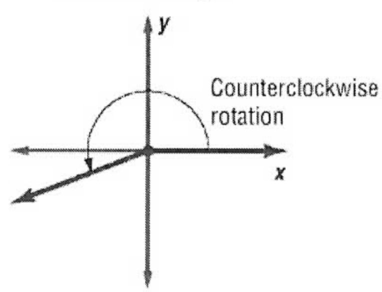
Today's Objectives

- Convert degree measures of angles to radian measures, and vice versa.
- Use angle measures to solve real-world problems.

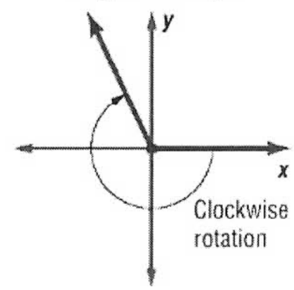
Angle in Standard Position



Positive Angle



Negative Angle



EXAMPLE 1 Convert Between DMS and Decimal Degree Form

1 deg = 60 min
1 min = 60 sec
1 deg = 3600 sec

A. Write 329.125° in DMS form.

$0.125° \cdot \frac{60 \text{ min}}{1°} = 7.5 \text{ min}$

$0.5 \text{ min} \times \frac{60 \text{ sec}}{1 \text{ min}} = 30 \text{ sec}$

329° 7' 30"

Answer: 329°7'30"

B. Write 35°12'7" in decimal degree form to the nearest thousandth.

~~$7'' = \frac{7}{60} = 0.11666 \text{ min}$~~

~~$12' = \frac{12}{60} = 0.2 \text{ degrees}$~~

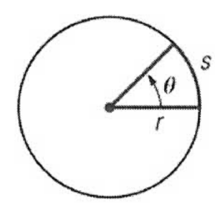
$35° + \frac{12}{60} + \frac{7}{3600}$

35.202°

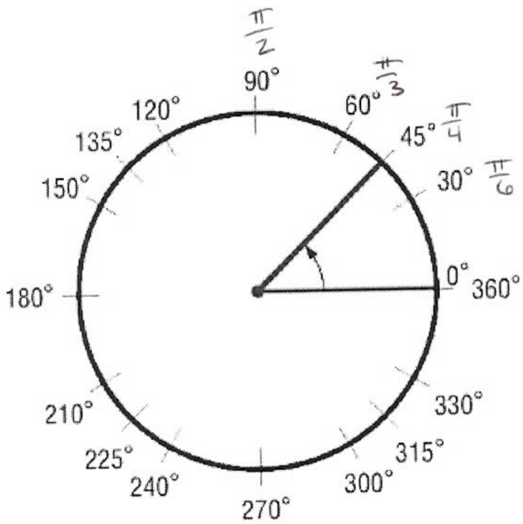
Answer: 35.202°

KeyConcept Radian Measure

Words	The measure θ in radians of a central angle of a circle is equal to the ratio of the length of the intercepted arc s to the radius r of the circle.
Symbols	$\theta = \frac{s}{r}$, where θ is measured in radians (rad)
Example	A central angle has a measure of 1 radian if it intercepts an arc with the same length as the radius of the circle.



$\theta = 1$ radian when $s = r$.



ratios

$\frac{\pi}{180}$ or $\frac{180}{\pi}$

$30° \cdot \frac{\pi}{180°} = \frac{\pi}{6}$

$45° \cdot \frac{\pi}{180} = \frac{\pi}{4}$

$60° \cdot \frac{\pi}{180} = \frac{\pi}{3}$

$90 \cdot \frac{\pi}{180} = \frac{\pi}{2}$

KeyConcept Degree/Radian Conversion Rules

1. To convert a degree measure to radians, multiply by $\frac{\pi \text{ radians}}{180°}$.
2. To convert a radian measure to degrees, multiply by $\frac{180°}{\pi \text{ radians}}$.

EXAMPLE 2 Convert Between Degree and Radian Measure

Answer

A. Write 135° in radians as a multiple of π. $\frac{3\pi}{4}$

$135° \cdot \frac{\pi}{180°} = \frac{3\pi}{4}$

B. Write −30° in radians as a multiple of π. $-\frac{\pi}{6}$

$-30° \cdot \frac{\pi}{180°} = -\frac{\pi}{6}$

C. Write $\frac{2\pi}{3}$ in degrees. 120°

$\frac{2\cancel{\pi}}{\cancel{3}_{1}} \cdot \frac{\cancel{180}^{60}}{\cancel{\pi}} = 120°$

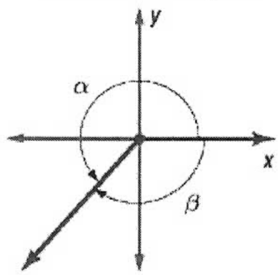
D. Write $-\frac{3\pi}{4}$ in degrees. −135°

$-\frac{3\cancel{\pi}}{\cancel{4}_{1}} \cdot \frac{\cancel{180}^{45}}{\cancel{\pi}} = -135°$

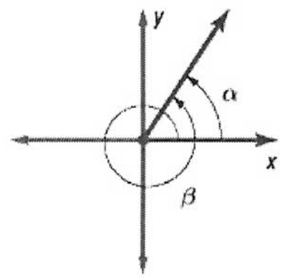
Think about a combination lock. When you go to the middle number, you actually go past the middle number the first time and stop when you reach the number the 2nd time.

Coterminal Angles are angles formed by either going backward toward an angle measure or going around the circle more than once, like the combination lock.

Positive and Negative Coterminal Angles



Positive Coterminal Angles

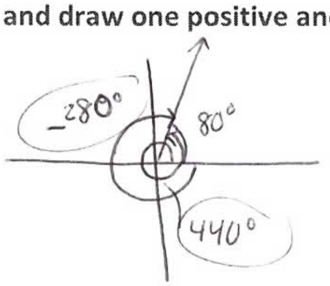


KeyConcept Coterminal Angles	
Degrees	Radians
$\theta + 360°n$	$\theta + 2\pi\, n$

$$where\ n\ is\ an\ integer$$

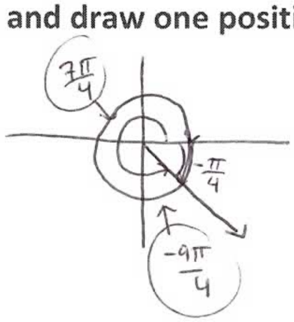
Find and draw one positive and one negative angle coterminal with 80°.

$$\begin{array}{r} 360 \\ -\ 80 \\ \hline 280 \end{array}$$



$80 + 360 = 440°$

Find and draw one positive and one negative angle coterminal with $-\frac{\pi}{4}$. $= -45°$



$2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

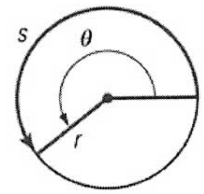
$-\frac{\pi}{4} - 2\pi = -\frac{9\pi}{4}$

KeyConcept Arc Length

If θ is a central angle in a circle of radius r, then the length of the intercepted arc s is given by

$$s = r\theta,$$

where $\boxed{\theta \text{ is measured in radians.}}$



A. Find the length of the intercepted arc in a circle with a central angle measure of $\frac{\pi}{3}$ and a radius of 4 inches. Round to the nearest tenth.

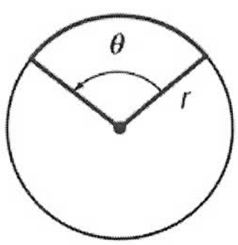
$S = 4 \cdot \frac{\pi}{3} = 4.2$ in

B. Find the length of the intercepted arc in a circle with a central angle measure of 125° and a radius of 7 centimeters. Round to the nearest tenth.

$S = \frac{25\pi}{36} \cdot 7 = 15.3$ cm

$125° \cdot \frac{\pi}{180°} = \frac{25\pi}{36}$

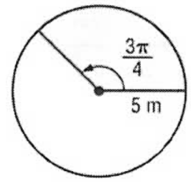
KeyConcept Area of a Sector



A **Sector** is a fraction of a circle. Therefore, the area is a fraction of the area of the circle.

Formula: $A = \left(\frac{\theta}{2\pi}\right)\pi r^2 \quad or \quad A = \left(\frac{\theta}{360°}\right)\pi r^2$

A. Find the area of the sector of the circle.

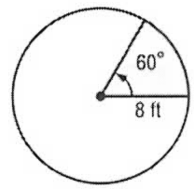


$$\frac{\frac{3\pi}{4}}{2\pi} \cdot \pi(5)^2$$

$$\frac{3\pi}{8} \cdot 25 = 29.45$$

or $\frac{75\pi}{8}$

B. Find the area of the sector of the circle.



$$\frac{60}{360} \cdot \pi(8)^2 = 33.5$$

or

$$\frac{1}{6} \cdot 64\pi = \frac{32\pi}{3}$$