5-Minute Check (over Chapter 3)

Use with Lesson **4-1**

1. Sketch and analyze the graph of $f(x) = -2^x + 3$. Describe its domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

1.


domain: $\mathbb{R}$
range: $y<3$
x-int: $0=-2^x+3$ y-int: $(0,2)$
$-3=-2^x$
$3=2^x$
$\log_2 3 = x$
$(\log_2 3, 0)$
asymptote: $y=3$
end behavior: $\lim\limits_{x\to-\infty} f(x)=3$
$\lim\limits_{x\to\infty} f(x)=-\infty$
decreasing: $(-\infty,\infty)$

2. Consider the table shown at the right.

x	y
0	2
1	7
2	22
3	72
4	237
5	778
6	2553

a. Make a scatter plot. (on the calculator)

b. Find an exponential function to model the data. $y=2.053(3.279)^x$

c. Find the value of the model at x = 20.
$y = 4.239 \times 10^{10}$

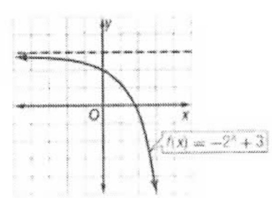
Standardized Test Practice

3. Solve $\log_3(5x) - \log_3(x+3) = \log_3 4$.

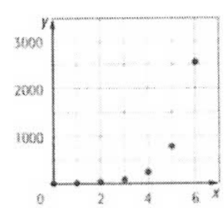
A -3 **B** $\frac{1}{6}$ **C** 3 **D** 12

$\log_3 \frac{5x}{x+3} = \log_3 4$
$\frac{5x}{x+3} = 4$
$5x = 4x+12$
$x=12$

ANSWERS

1.


$D = (-\infty, \infty)$; $R = (-\infty, 3)$; intercept: (0, 2); asymptote: $y = 3$; $\lim\limits_{x\to-\infty} f(x) = 3$; $\lim\limits_{x\to\infty} f(x) = -\infty$; decreasing on $(-\infty, \infty)$

2a.


2b. $y = 2.05(3.28)^x$

2c. 4.26×10^{10}

3. D

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Answers: 8. $y=1$, 9. $x=-5$, 10. $x=-4, x=2$, 11. $x=3, x=5, y=0$ 12. & 13. no asymptotes,

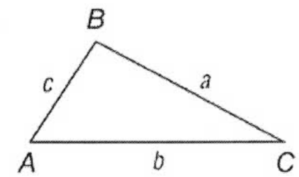
Today's Objective:

no right angle

- Solve oblique triangles using the Law of Sines or the Law of Cosines.

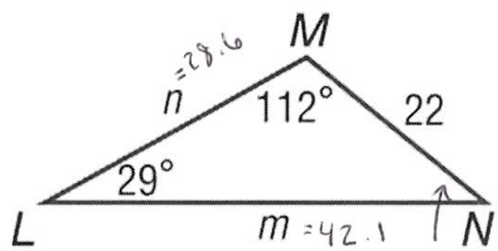
KeyConcept **Law of Sines**

If $\triangle ABC$ has side lengths a, b, and c representing the lengths of the sides opposite the angles with measures A, B, and C, then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.



Example 1:

Solve ΔLMN.



$180 - 112 - 29 = 39°$

$$\frac{\sin 29}{22} = \frac{\sin 112}{m}$$

$$\frac{m \sin 29}{\sin 29} = \frac{22 \sin 112}{\sin 29}$$

$$m = 42.1$$

$$\frac{\sin 29}{22} = \frac{\sin 39}{n}$$

$$\frac{n \sin 29}{\sin 29} = \frac{22 \sin 39}{\sin 29}$$

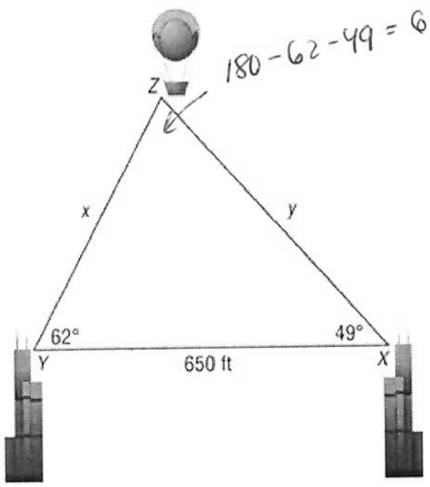
$$n = 28.6$$

Answer: $N = 39°$, $m \approx 42.1$, $n \approx 28.6$

Example 2:

The angle of elevation from the top of a building to a hot air balloon is 62º. The angle of elevation to the hot air balloon from the top of a second building that is 650 feet due east is 49º. Find the distance from the hot air balloon to each building.

Can you draw a figure showing the situation?



$180-62-49=69°$

$\frac{\sin 69}{650}=\frac{\sin 62}{y}$

$\frac{y\sin 69}{\sin 69}=\frac{650\sin 62}{\sin 69}$

$y=614.7$

$\frac{\sin 69}{650}=\frac{\sin 49}{x}$

$\frac{x\sin 69}{\sin 69}=\frac{650\sin 49}{\sin 69}$

$x=525.5$

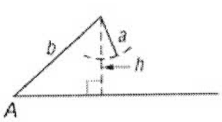
Answer: about 525.5 ft ; 614.7 ft

side, side, angle (non-included)

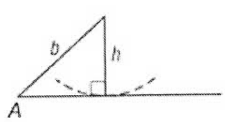
KeyConcept The Ambiguous Case (SSA)

Consider a triangle in which a, b, and A are given. For the acute case, $\sin A=\frac{h}{b}$, so $h=b\sin A$.

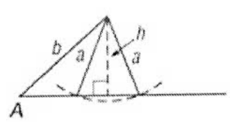
A is Acute. ($A<90°$)



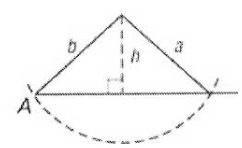
$a<b$ and $a<h$
no solution



$a<b$ and $a=h$
one solution

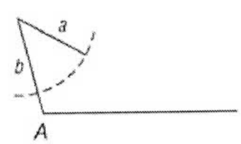


$a<b$ and $a>h$
two solutions

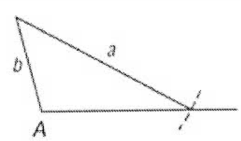


$a\geq b$
one solution

A is Right or Obtuse. ($A\geq 90°$)



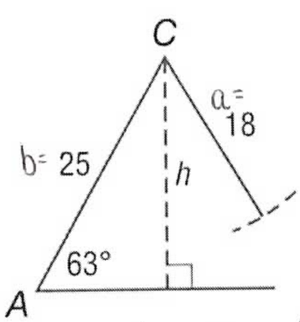
$a\leq b$, no solution



$a>b$, one solution

Example 3: Find all solutions for the given triangle, if possible. $A=63°$, $a=18$, $b=25$

$a<b$



1. Find h

$\sin 63=\frac{h}{25}$

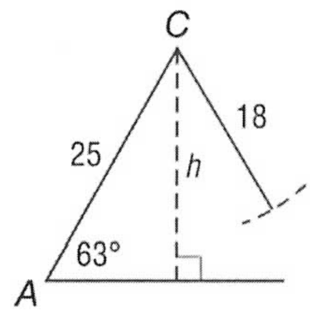
$25\sin 63=h$

$22.3=h$

$a<h$

$\boxed{\text{no solution}}$ does not make a triangle

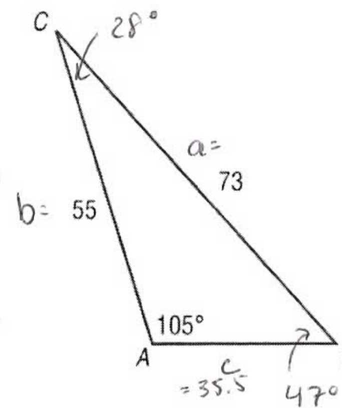
triangle, if possible. $A = 63°$, $a = 18$, $b = 25$



Because $a < h$, no triangle can be formed with sides $a = 18$, $b = 25$, and $A = 63°$. Therefore, the problem has no solution.

Example 4:

Find all solutions for the given triangle, if possible. $A = 105°$, $a = 73$, $b = 55$



$a > b$ one solution

1. $\frac{\sin 105}{73} = \frac{\sin B}{55}$

$\frac{73 \sin B}{73} = \frac{55 \sin 105}{73}$

$\sin B = 0.72775$

$\sin^{-1}(0.72775) = B$

$B = \cancel{46}.\ 47°$

2. $C = 180 - 105 - 47$

$C = 28°$

3. $\frac{\sin 105}{73} = \frac{\sin 28}{c}$

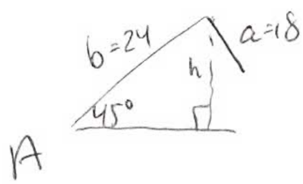
$\frac{c \sin 105}{\sin 105} = \frac{73 \sin 28}{\sin 105}$

$c = 35.5$

Answer: $B \approx 47°$, $C \approx 28°$, $c \approx 35.5$

Example 5: Find two triangles for which $A = 45°$, $a = 18$, and $b = 24$.

$a < b$



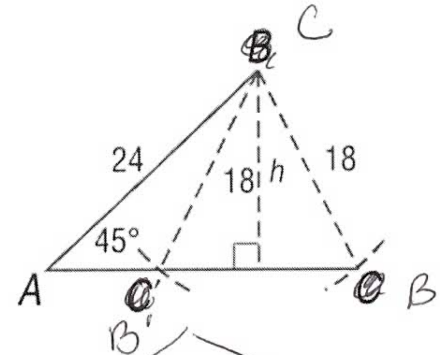
1. Find h

$\sin 45 = \frac{h}{24}$

$24 \sin 45 = h$

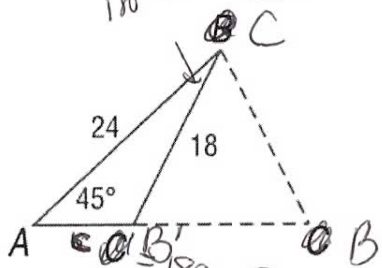
$16.97 = h$

$a > h$ 2 solutions



1st solution

$180-45-109 = 26°$



$B' = 180 - B$
$= 180 - 71$
$= 109°$

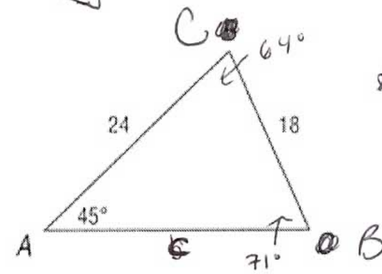
* makes sense, this is an obtuse angle

$\frac{\sin 45}{18} = \frac{\sin 26}{c}$

$\frac{c \sin 45}{\sin 45} = \frac{18 \sin 26}{\sin 45}$

$c = 11.2$

2nd solution



* start here

$\frac{\sin 45}{18} = \frac{\sin B}{24}$

$\frac{18 \sin B}{18} = \frac{24 \sin 45}{18}$

$\sin B = 0.9428$

$\sin^{-1}(0.9428) = B$

$71° = B$

$C = 180 - 45 - 71 = 64°$

$\frac{\sin 45}{18} = \frac{\sin 64}{c}$

$\frac{c \sin 45}{\sin 45} = \frac{18 \sin 64}{\sin 45}$

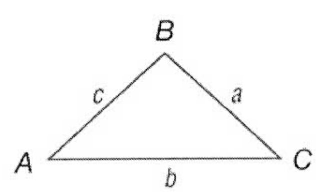
$c = 22.9$

~~(Answer: B ≈ 64°, C ≈ 71°, b ≈ 23.0; B ≈ 26°, C ≈ 109°, b ≈ 11.0~~

KeyConcept Law of Cosines

In $\triangle ABC$, if sides with lengths a, b, and c are opposite angles with measures A, B, and C, respectively, then the following are true.

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$



Example 6: LANDSCAPING A triangular area of lawn has a

sprinkler located at each vertex. If the sides of the lawn are *a* = 19 feet, *b* = 24.3 feet, and *c* = 21.8 feet, what angle of sweep should each sprinkler be set to cover?

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

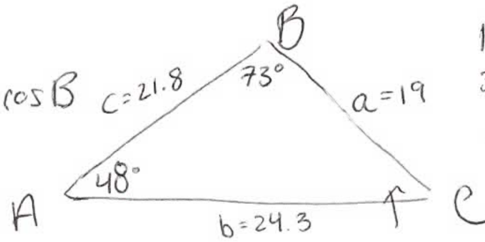
$24.3^2 = 19^2 + 21.8^2 - 2(19)(21.8)\cos B$

$590.49 = 836.24 - 828.4\cos B$

$-245.75 = -828.4\cos B$

$0.294656 = \cos B$

$73° = B$



$19^2 = 24.3^2 + 21.8^2 - 2(21.8)(24.3)\cos A$

$361 = 1065.73 - 1059.48\cos A$

SSS = law of cosines

$-704.73 = -1059.48\cos A$

$0.66516 = \cos A$

$48.° = A$

$180 - 48 - 73 = 59°$

Answer: A ≈ 48°, B ≈ 73°, C ≈ 59°.

Example 7: Solve Δ*ABC*.

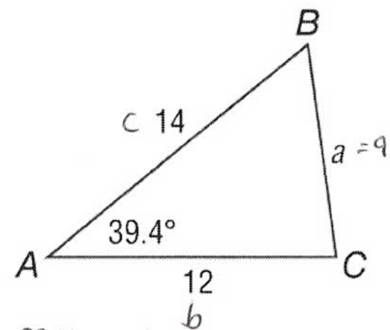
$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 12^2 + 14^2 - 2(12)(14)\cos(39.4)$

$a^2 = 340 - 336\cos(39.4)$

$a^2 = 80.3615$

$a = 9$



$\frac{\sin(39.4)}{9} = \frac{\sin C}{14}$

$\frac{9\sin C}{9} = \frac{14\sin(39.4)}{9}$

$\sin C = 0.9874$

$C = 81°$

$\frac{\sin 39.4}{9} = \frac{\sin B}{12}$

$\frac{9\sin B}{9} = \frac{12\sin 39.4}{9}$

$\sin B = 0.8463$

$B = 57.8°$

SAS = law of cosines

Answer: $a \approx 9.0$, $B \approx 58°$, $C \approx 83°$

Assignment: Page 298 (1-5,11-17,19-23,27-33 odds only)
* Check your answers before class tomorrow.